

Cosmic inferences from the morphology of the Universe

Intensive Report

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Outline

Introduction to " Cosmic inferences from the morphology of the Universe "

Imprint of massive neutrinos on Persistent Homology of large-scale structure

Probing the anisotropy and non-Gaussianity in redshift space through the derivative of excursion set moments

Morphology of CMB fields: effect of weak gravitational lensing

Morphology of LSS in the presence of primordial non-Gaussianity: Scale dependent bias of critical Points

Testing Cosmological Principles

Morphology of LSS: Constraining the Modified Gravity

Simulation Based Inference

Main Topics of Thesis

In Progress

In Progress

Relevant Topics

In Progress

What is meant by morphology?

 δ_1

-0.9	2.0	5.5	1.4	1.5	1.9	6.0	
1.3	1.2	-1.3	1.5	0.2	-0.9	4.2	
5.7	1.7	4.5	4.6	4.2	1.5	0.0	
0.5	0.0	5.2	2.0	4.1	0.0	-0.2	
2.1	2.4	4.5	4.3	4.9	3.0	3.4	
2.4	-0.9	3.0	0.8	2.7	0.0	5.9	
2.1	2.4	2.5	3.9	5.7	3.5	2.6	

 δ_2

1.3	1.2	-1.3	1.5	0.2	-0.9	4.2	
0.5	0.0	5.2	2.0	4.1	0.0	-0.2	
5.7	1.7	4.5	4.6	4.2	1.5	0.0	
2.1	2.4	4.5	4.3	4.9	3.0	3.4	
2.1	2.4	2.5	3.9	5.7	3.5	2.6	
2.4	-0.9	3.0	0.8	2.7	0.0	5.9	
-0.9	2.0	5.5	1.4	1.5	1.9	6.0	

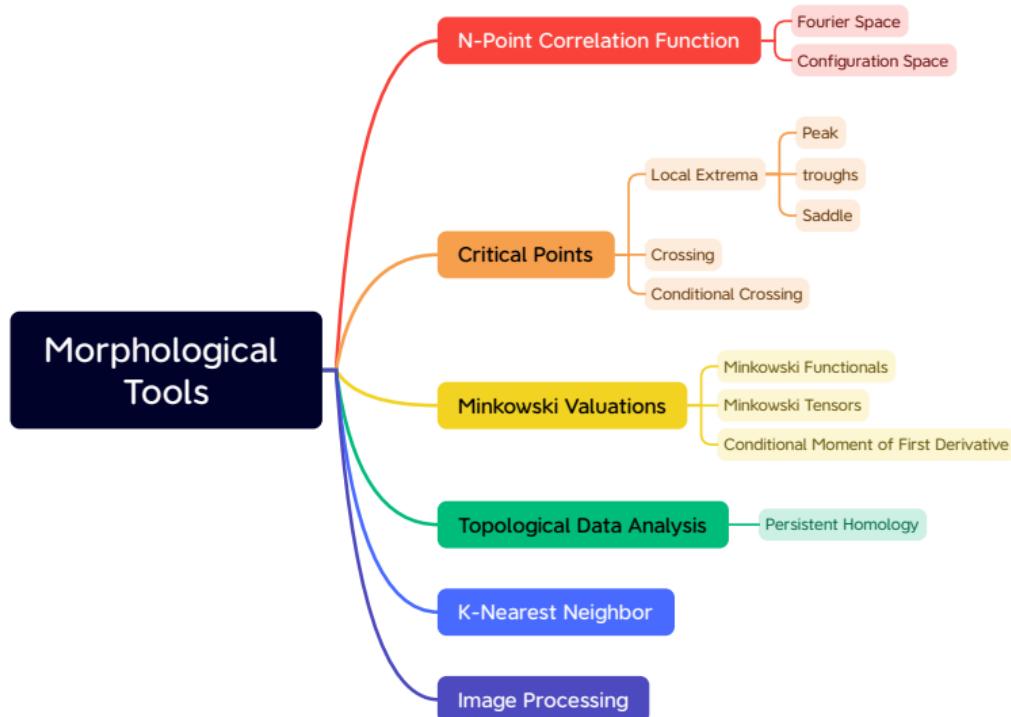
 δ_3

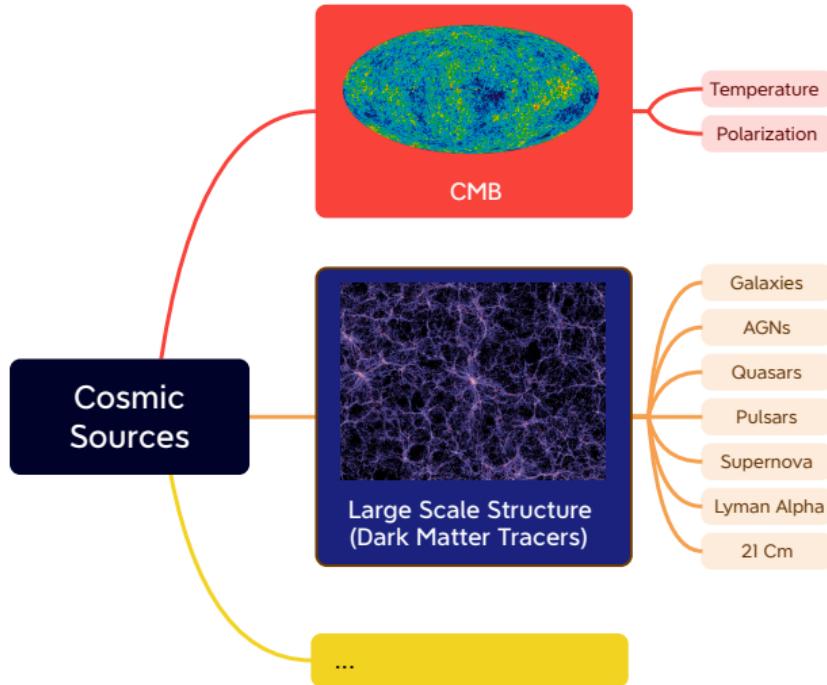
2.1	2.4	2.5	3.9	5.7	3.5	2.6	
5.7	1.7	4.5	4.6	4.2	1.5	0.0	
0.5	0.0	5.2	2.0	4.1	0.0	-0.2	
-0.9	2.0	5.5	1.4	1.5	1.9	6.0	
2.4	-0.9	3.0	0.8	2.7	0.0	5.9	
1.3	1.2	-1.3	1.5	0.2	-0.9	4.2	
2.1	2.4	4.5	4.3	4.9	3.0	3.4	

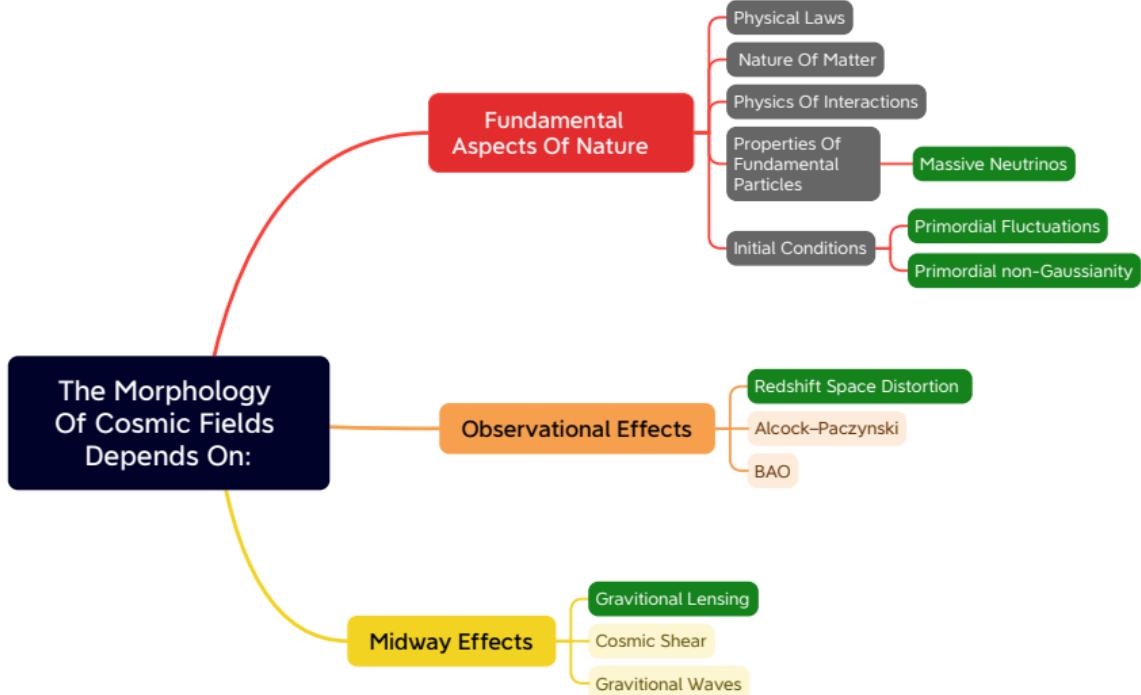
 δ_4

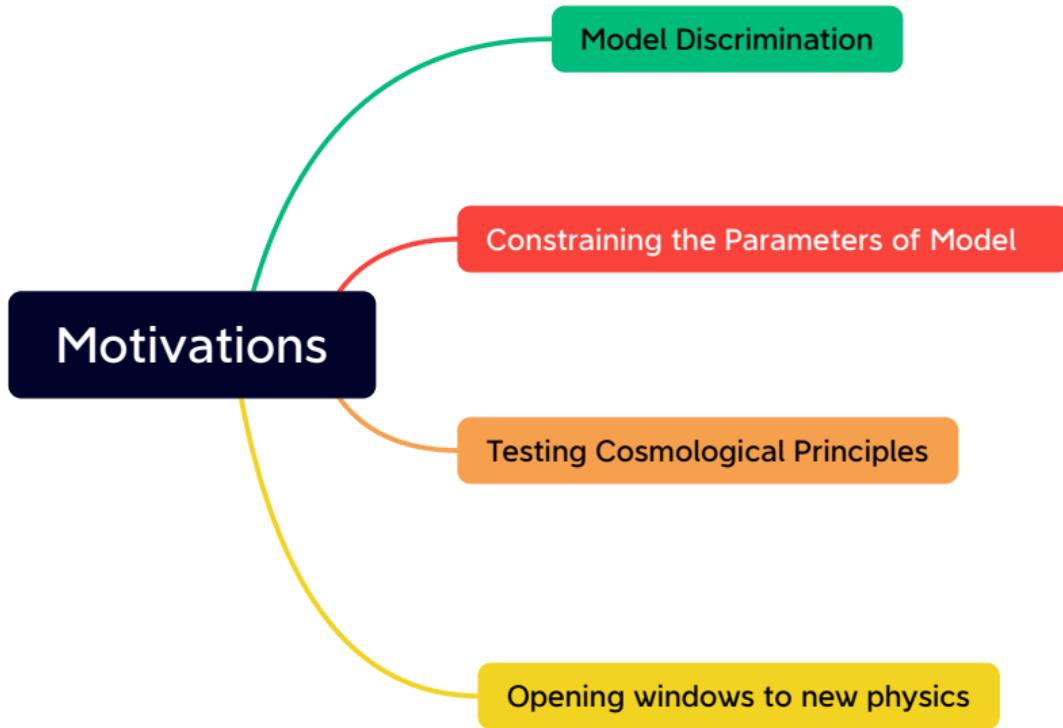
-0.9	2.0	5.5	1.4	1.5	1.9	6.0	
2.4	-0.9	3.0	0.8	2.7	0.0	5.9	
0.5	0.0	5.2	2.0	4.1	0.0	-0.2	
5.7	1.7	4.5	4.6	4.2	1.5	0.0	
2.1	2.4	2.5	3.9	5.7	3.5	2.6	
2.1	2.4	4.5	4.3	4.9	3.0	3.4	
1.3	1.2	-1.3	1.5	0.2	-0.9	4.2	

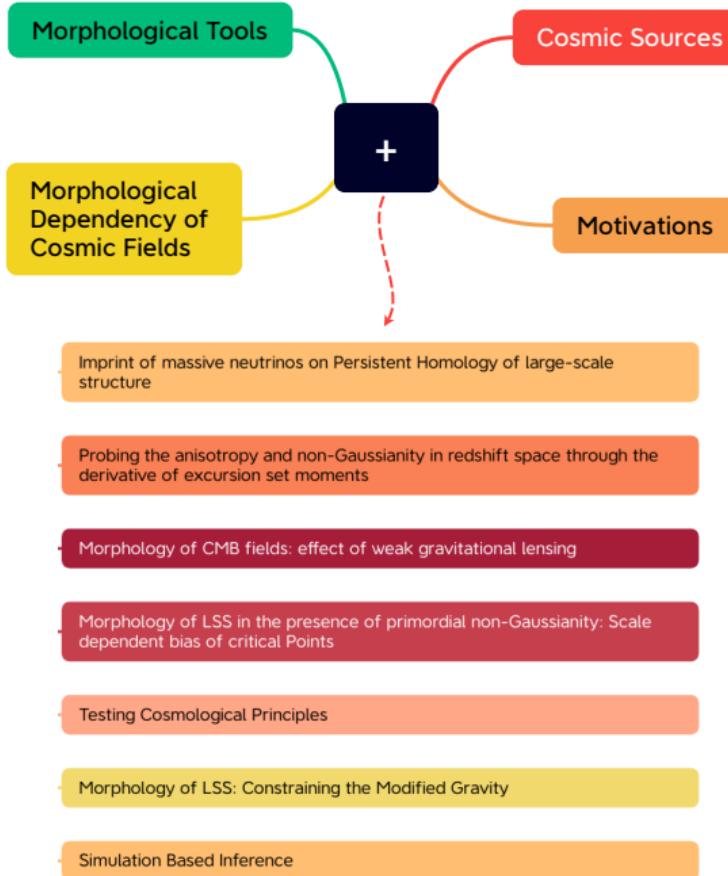
$$\text{PDF}(\delta_1) = \text{PDF}(\delta_2) = \text{PDF}(\delta_3) = \text{PDF}(\delta_4)$$





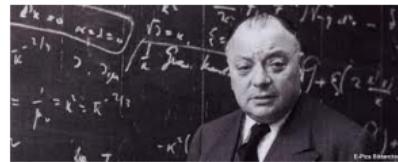






Imprint of massive neutrinos on Persistent Homology of large-scale structure

Neutrino Physics



Neutrinos were first theorized in 1930 by Wolfgang Pauli to explain the continuous energy spectrum of protons and electrons in β -decays

The first neutrino detection dates back in 1956

According to the Standard Model of particle physics:

Neutrinos Come In Three Different Flavors

Only Interact Via The Weak Nuclear Force

Are Massless

Confirmed by the latest experimental data (Tanabashi et al. (2018))

There are wellmotivated physical models where neutrinos acquire mass (e.g. Gonzalez-Garcia & Nir (2003); Mohapatra & Smirnov (2006))

Massive Neutrino?

Neutrino oscillations: One way to detect whether neutrinos are massive

To explain the measured flux of electron neutrinos from the Sun

The measured neutrinos are not exact eigenstates of the Standard Model Lagrangian

neutrino oscillations only depend on the difference of square masses between different species

Neutrinos are a linear combination of three mass states

$$M_\nu \equiv \sum_{i=1}^3 m_i$$

Atmospheric neutrino:

$$|\Delta m_{31}^2| = |m_3^2 - m_1^2| \simeq |\Delta m_{32}^2| = |m_3^2 - m_2^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2$$

Solar neutrino:

$$\Delta m_{21}^2 = m_2^2 - m_1^2 \simeq 7.55 \times 10^{-5} \text{ eV}^2$$

$$M_{\nu, \min} \approx \begin{cases} 0.058 \text{ eV} & (\text{NH}) \\ 0.1 \text{ eV} & (\text{IH}) \end{cases}$$

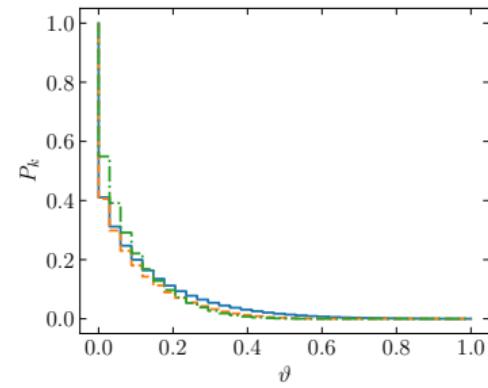
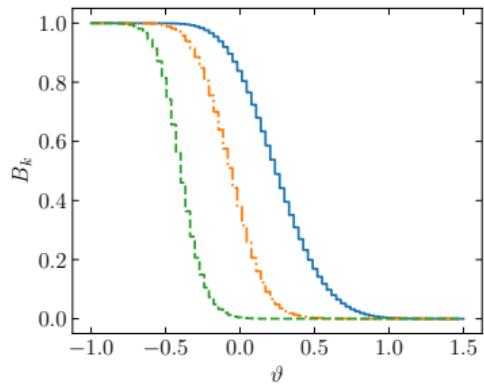
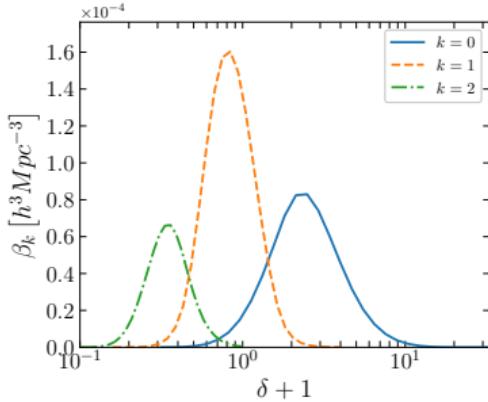
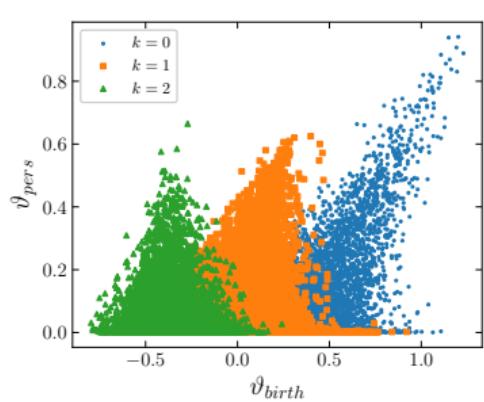
Cosmology is mainly sensitive to the **sum of the three neutrino masses** (M_ν)

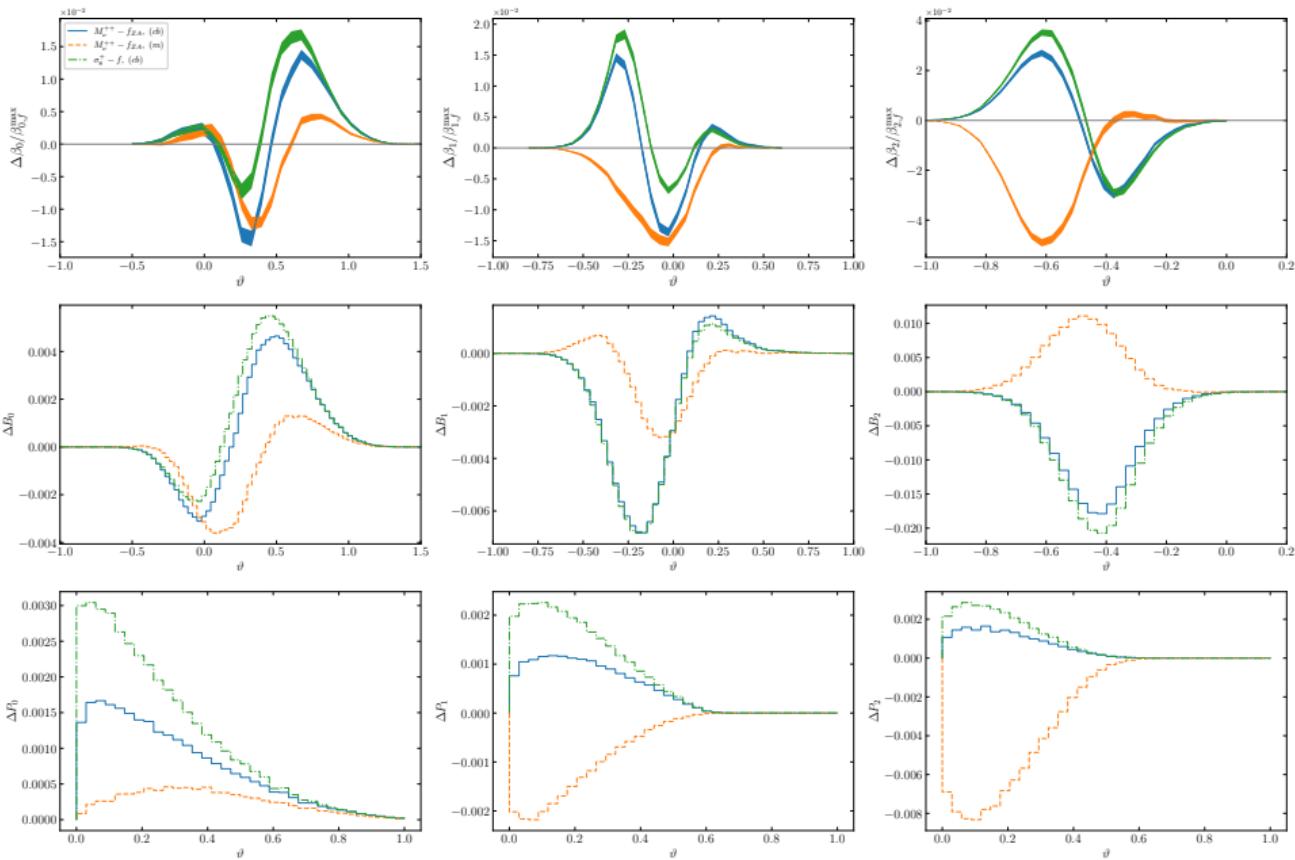
Upper limit from (CMB + BAO + CMB Lensing): $M_\nu \leq 0.115$ eV

$$\beta_k(\vartheta) = \sum_{i=1}^{n_k} \Theta\left(\vartheta_{(i),birth}^{(k)} - \vartheta\right) \Theta\left(\vartheta - \vartheta_{(i),death}^{(k)}\right)$$

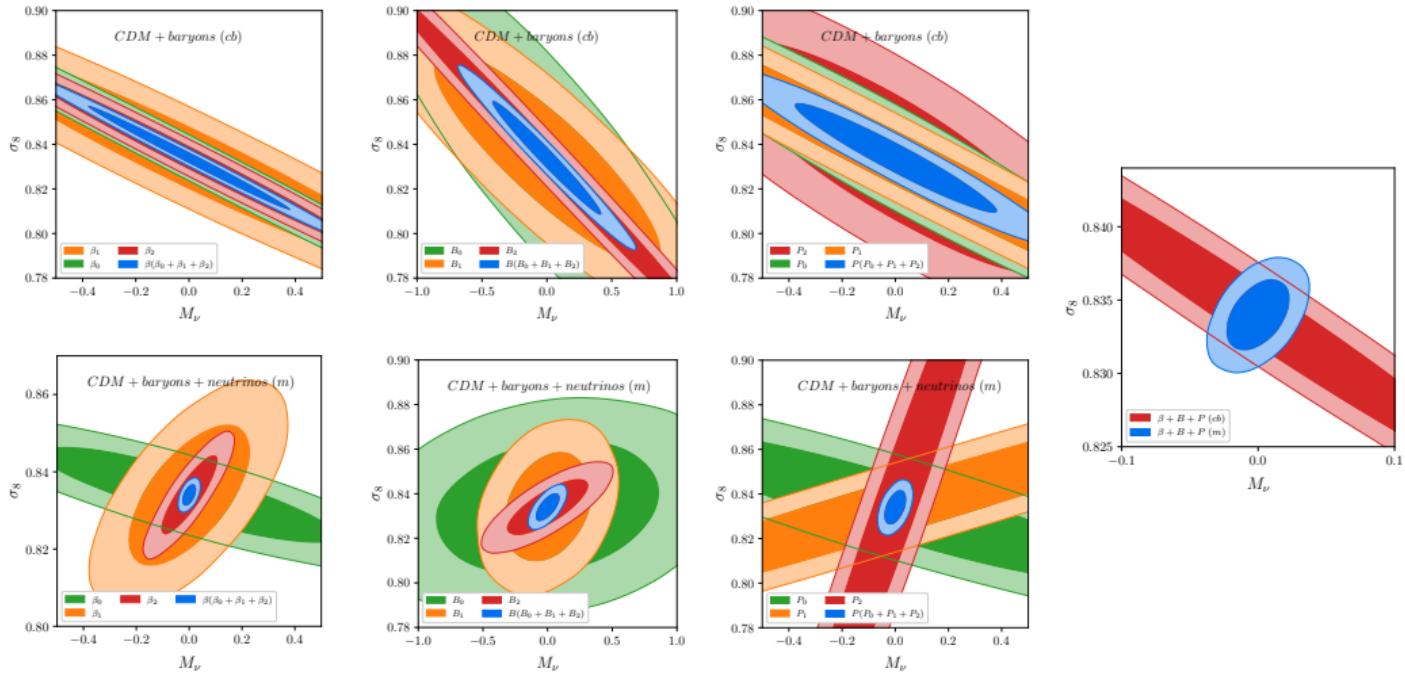
$$B_k(\vartheta) \equiv \sum_{i=1}^{n_k} \Theta\left(\vartheta - \vartheta_{(i),birth}^{(k)}\right)$$

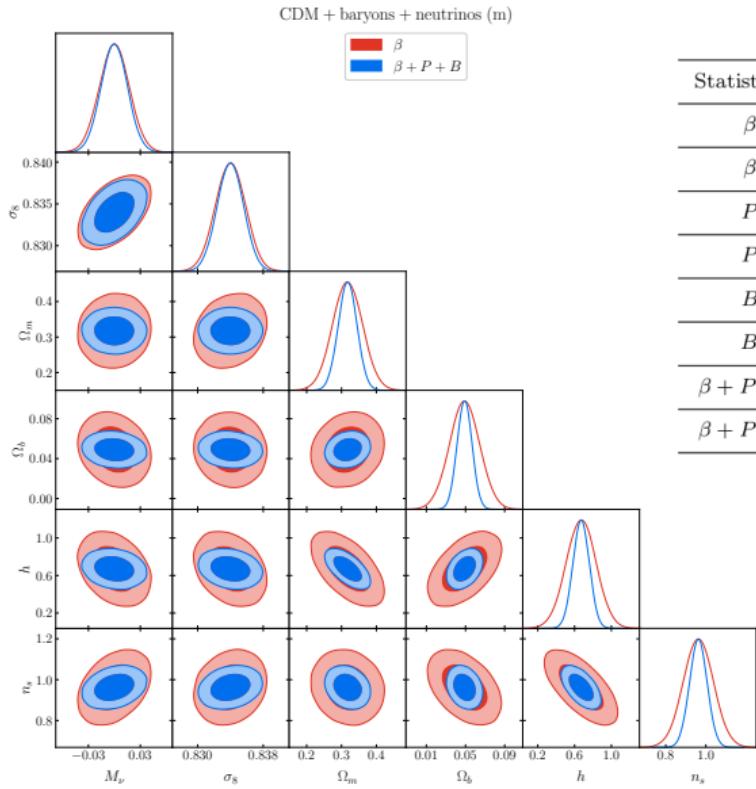
$$P_k(\vartheta) \equiv \sum_{i=1}^{n_k} \Theta\left(\vartheta - \vartheta_{(i),pers}^{(k)}\right)$$





Fisher Forecast



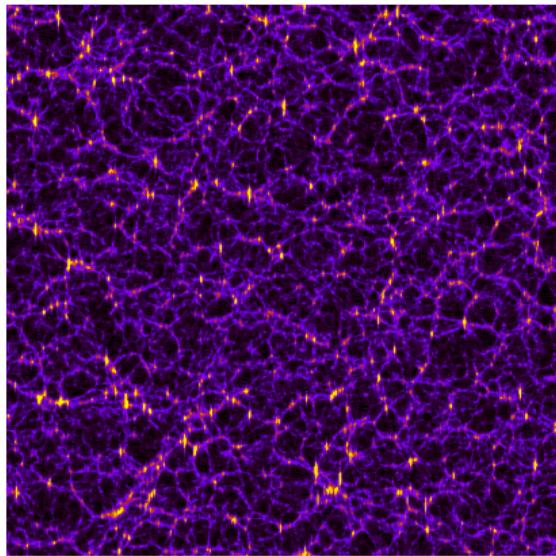
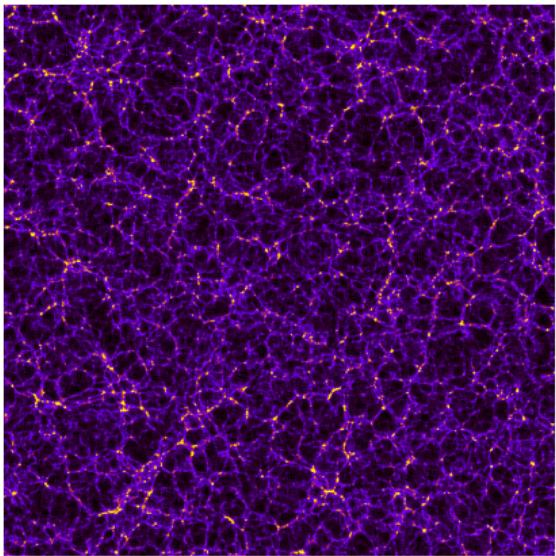


Statistics (field)	M_ν (eV)	σ_8	Ω_m	Ω_b	h	n_s
β (cb)	0.2504	0.0153	0.0442	0.0156	0.1546	0.0699
β (m)	0.0172	0.0018	0.0427	0.0152	0.1617	0.0747
P (cb)	0.2511	0.0162	0.0559	0.0163	0.1640	0.1128
P (m)	0.0269	0.005	0.0564	0.0163	0.1650	0.1224
B (cb)	0.2779	0.0168	0.062	0.0163	0.0138	0.1694
B (m)	0.0610	0.0041	0.0615	0.0139	0.1709	0.1473
$\beta + P + B$ (cb)	0.1242	0.0077	0.027	0.0075	0.0878	0.0423
$\beta + P + B$ (m)	0.0152	0.0015	0.0267	0.0075	0.0886	0.0436

Probing the anisotropy and non-Gaussianity in redshift space through the derivative of excursion set moments

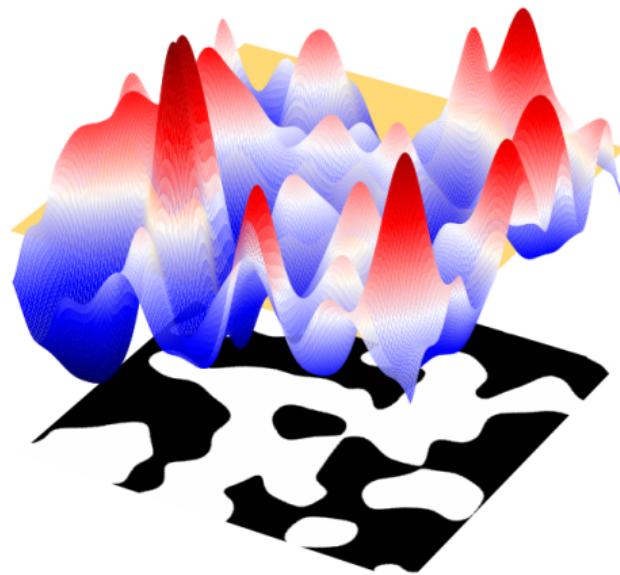
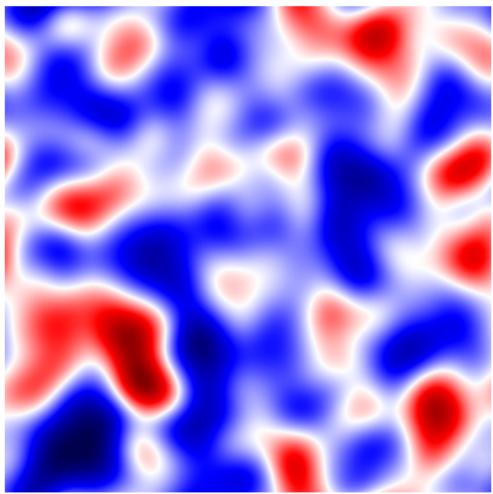
see [arXiv:2308.03086](https://arxiv.org/abs/2308.03086)

Redshift Space distortion



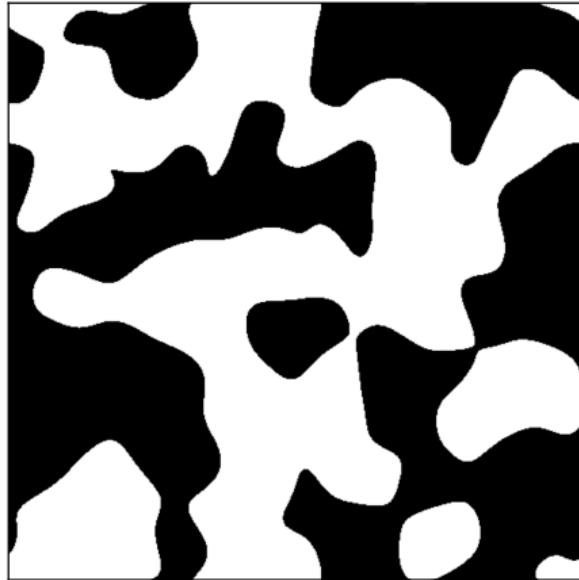
Minkowski Valuations

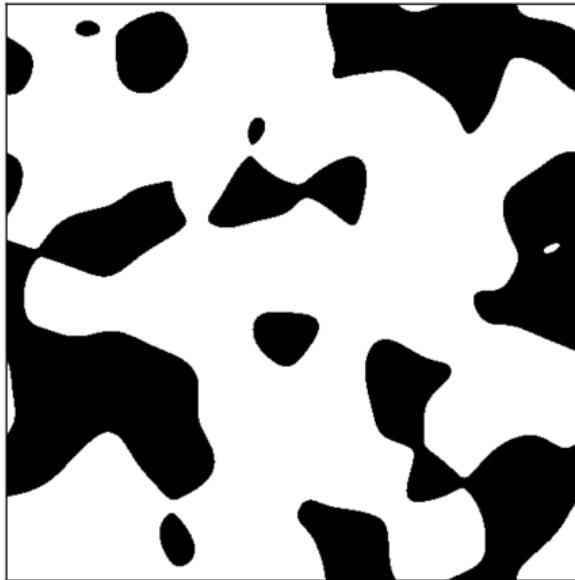
$$\mathcal{Q}_\vartheta = \{\mathbf{r} \in \mathcal{M} \mid \delta(\mathbf{r}) \geq \vartheta\}$$

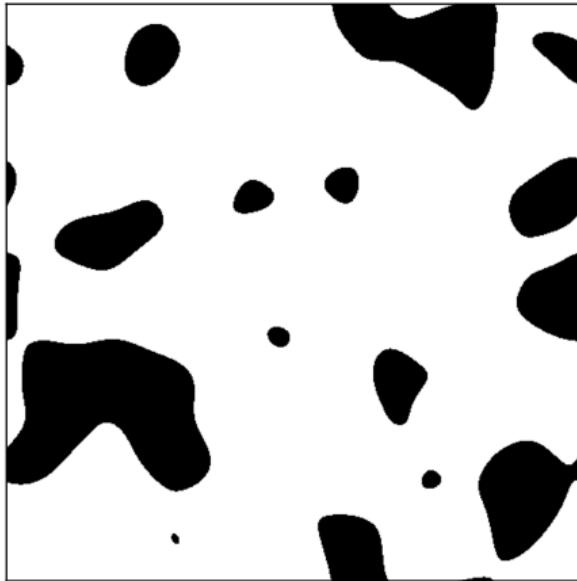


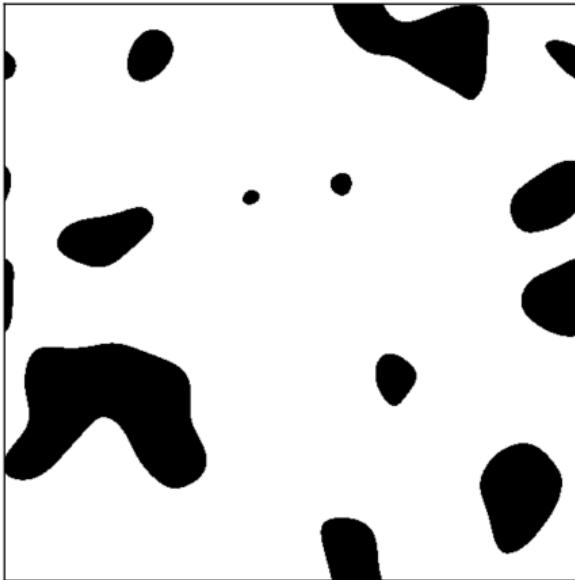


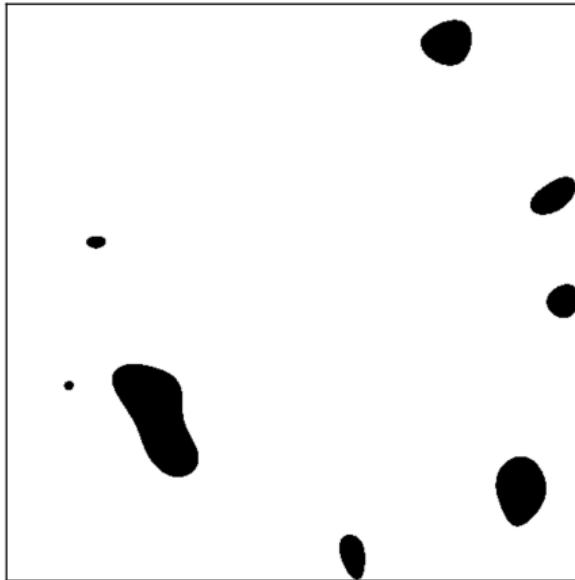












General Form

$$\Xi \equiv \int_{Q_\vartheta} dV_d \mathcal{G}(s_\nu; \vec{r}, \delta, \nabla \delta, \dots)$$

$$\mathcal{W}_\nu^{(p,q)} \equiv \int_{\partial Q_\vartheta} dA_d s_\nu \underbrace{\vec{r} \otimes \vec{r} \otimes \dots \otimes \vec{r}}_{p-times} \\ \otimes \underbrace{\frac{\nabla \delta}{|\nabla \delta|} \otimes \frac{\nabla \delta}{|\nabla \delta|} \otimes \dots \otimes \frac{\nabla \delta}{|\nabla \delta|}}_{q-times}$$

Minkowski Tensors

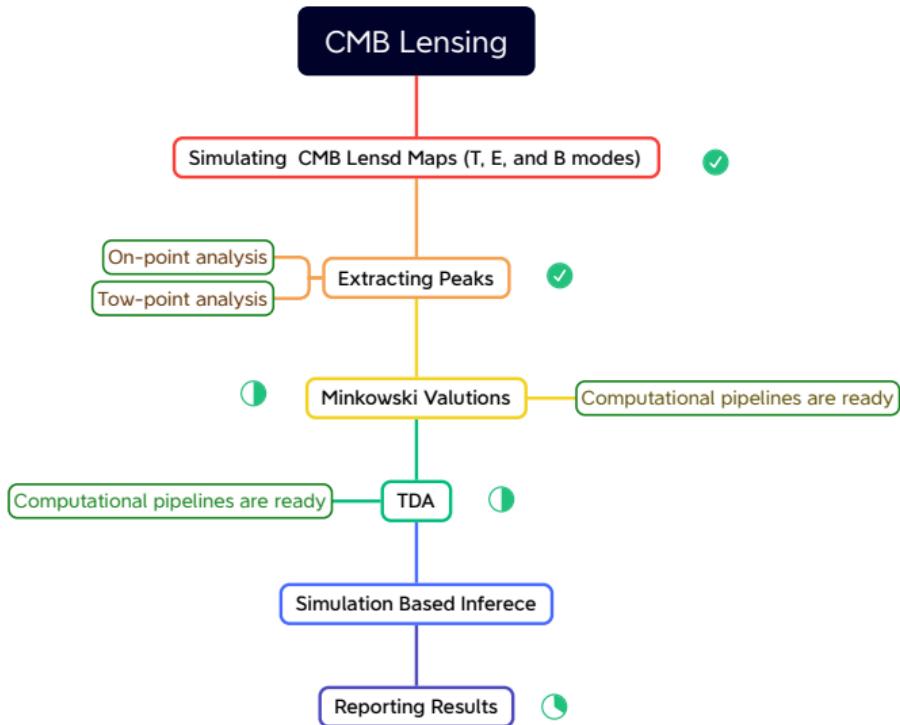
$$N_{cr}^{(r,s)}(\vartheta, i; n) = \frac{1}{V} \int_V dV \delta_D \left(\delta^{(r,s)} - \vartheta \sigma_0^{(r,s)} \right) \left| \delta_{,i}^{(r,s)} \right| \\ = \frac{1}{V} \int_{\partial Q_v} dA \frac{\left| \delta_{,i}^{(r,s)} \right|}{\left| \nabla \delta^{(r,s)} \right|}$$

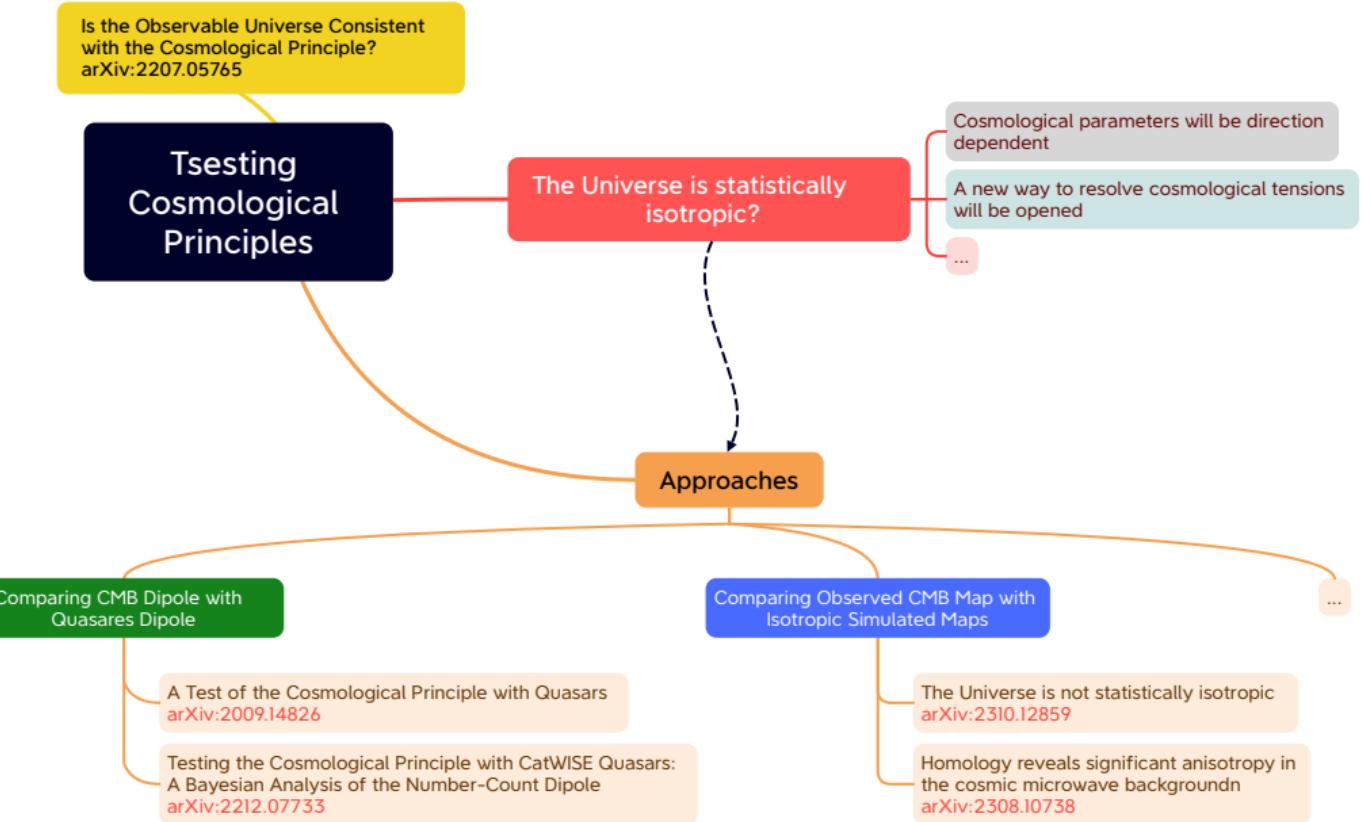
Crossing Statistics

$$N_{cmd}^{(r,s)}(\vartheta, i; n) \equiv \frac{1}{V} \int_V dV \delta_D \left(\delta^{(r,s)} - \vartheta \sigma_0^{(r,s)} \right) \left(\delta_{,i}^{(r,s)} \right)^n \\ = \frac{1}{V} \int_{\partial Q_v} dA \frac{\left(\delta_{,i}^{(r,s)} \right)^n}{\left| \nabla \delta^{(r,s)} \right|}$$

Conditional Moments of the First Derivative (cmd)

Morphology of CMB fields: effect of weak gravitational lensing

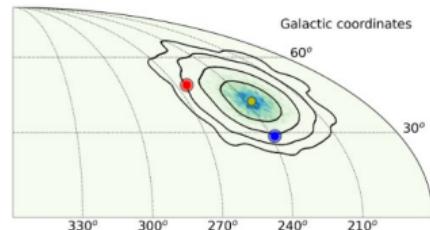
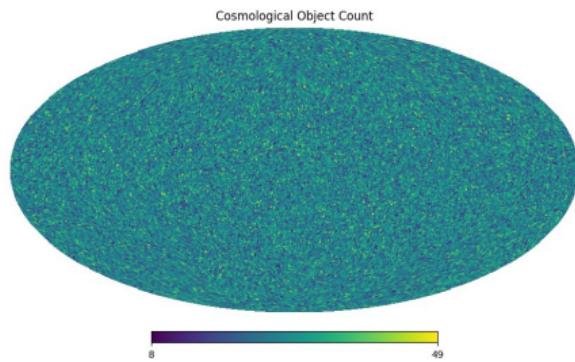




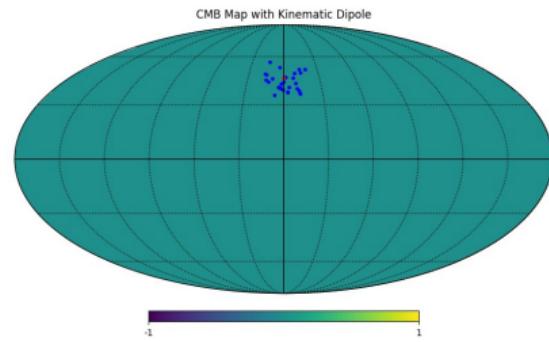
$$N_{\text{obs}}(\mathbf{n}, > S) = N_{\text{rest}}(\mathbf{n}, > S) \left[1 + A \mathbf{n} \cdot \hat{\boldsymbol{\beta}} \right]$$

$$A = [2 + x(1 + \alpha)] \beta$$

$$\min \sum_p \frac{[N_p(\mathbf{n}, > S) - \bar{N}(> S) (1 + A \cos \theta_p)]^2}{\bar{N}(> S) (1 + A \cos \theta_p)}$$

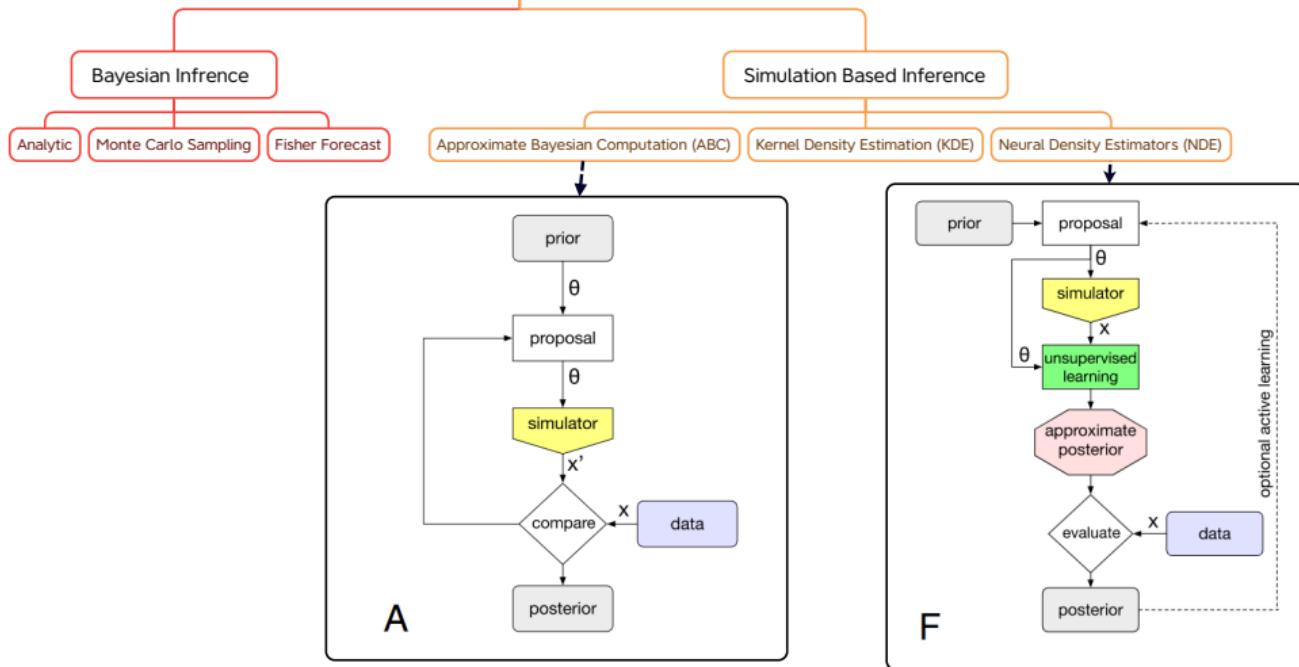


Bengaly, Carlos AP, et al. MNRAS (2019): 1350-1357



Data Modeling: Estimating the posterior function

$$P(\theta|\mathcal{D}) \sim \mathcal{L}(\mathcal{D}|\theta) P(\theta)$$



Cranmer, Kyle, Johann Brehmer, and Gilles Louppe. Proceedings of the National Academy of Sciences 117.48 (2020): 30055-30062.