Kolmogrov Complexity the Detect Phase Tranistion

By :

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COMPLEXITY

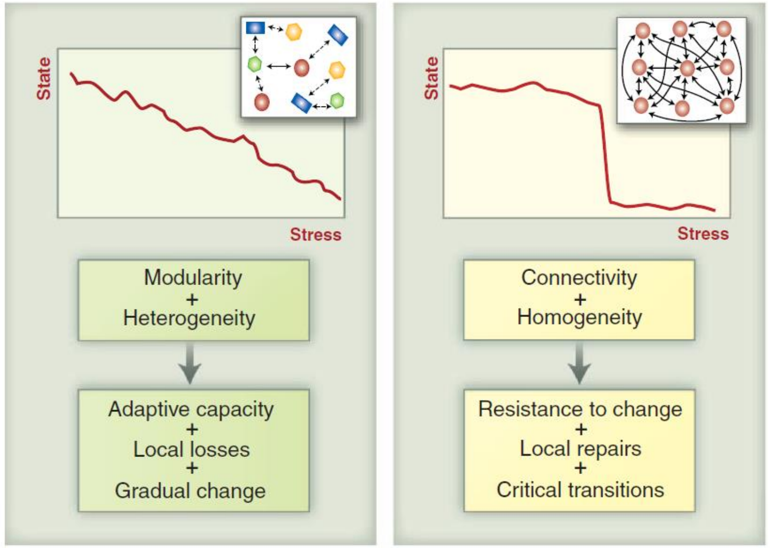
***Complexity** science, also called **complex systems** science, studies how a **large collection** of **components** – locally interacting with each other at small scales – can spontaneously self-organize to exhibit non-trivial global structures and behaviors at larger scales, often without external intervention, central authorities or leaders. The properties of the collection may not be understood or predicted from the full knowledge of its constituents alone. Such a collection is called a complex system and it requires new mathematical frameworks and scientific methodologies for its investigation.*

There's no love in a carbon atom, no hurricane in a water molecule, no financial collapse in a dollar bill.

-peter dodds

INTERACTIONS

Complex systems are often characterized by ***many components*** *that interact in multiple ways among each other and potentially with their environment too. These components form networks of interactions, sometimes with just a few components involved in many interactions. Interactions may generate novel information that make it difficult to study components in isolation or to completely predict their future. In addition, the components of a system can also be whole new systems, leading to systems of systems, being interdependent on one another. The main challenge of complexity science is not only to see the parts and their connections but also to understand how these connections give rise to the whole.*



Mitchell, Melanie. Complexity: A Guided Tour. Oxford University Press, 2009.

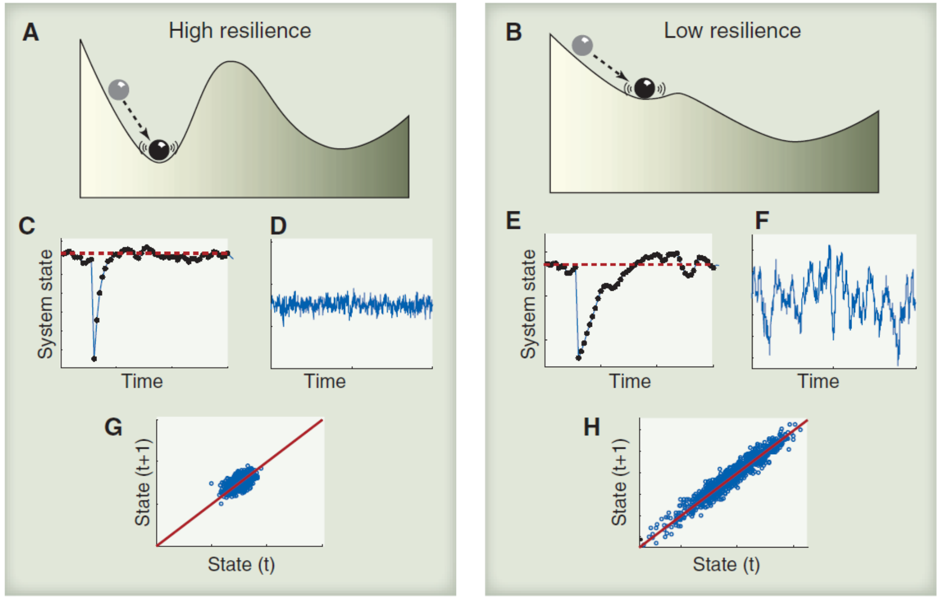
DYNAMICS

Systems can be analyzed in terms of the changes of their states over time. A state is described in sets of variables that best characterize the system. As the system changes its state from one to another, its variables also change, often responding to its environment. This change is called linear if it is directly proportional to time, the system's current state, or changes in the environment, or non-linear if it is not proportional to them. Complex systems are typically non-linear, changing at different rates depending on their states and their environment. They also may have stable states at which they can stay the same even if perturbed, or unstable states at which the systems can be disrupted by a small perturbation. In some cases, small environmental changes can completely change the system behavior, known as bifurcations, phase transitions, or "tipping points."

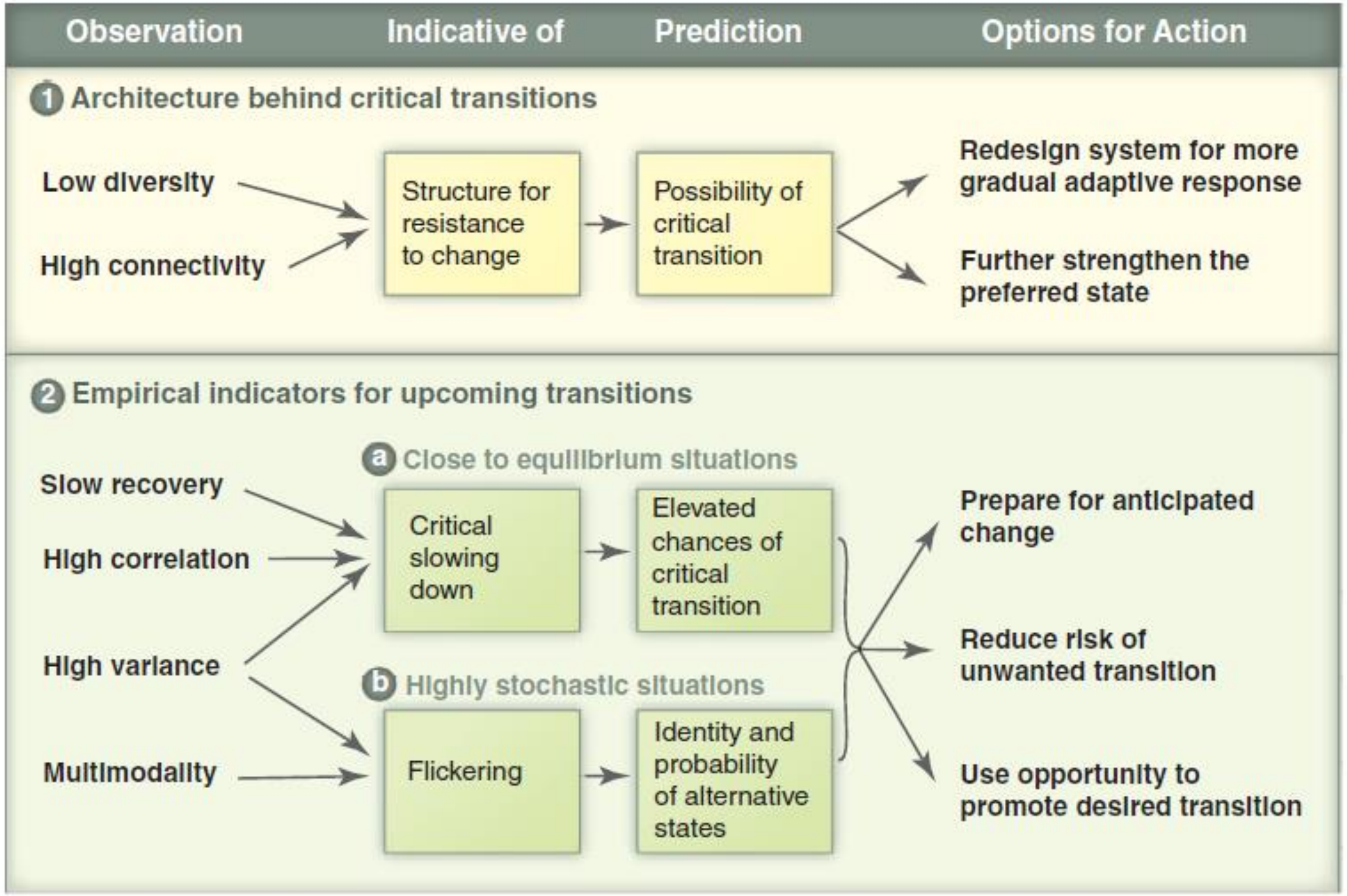
Strogatz, Steven H. Nonlinear Dynamics and Chaos. CRC Press, 1994.

RESILIENCE & ADAPTATION

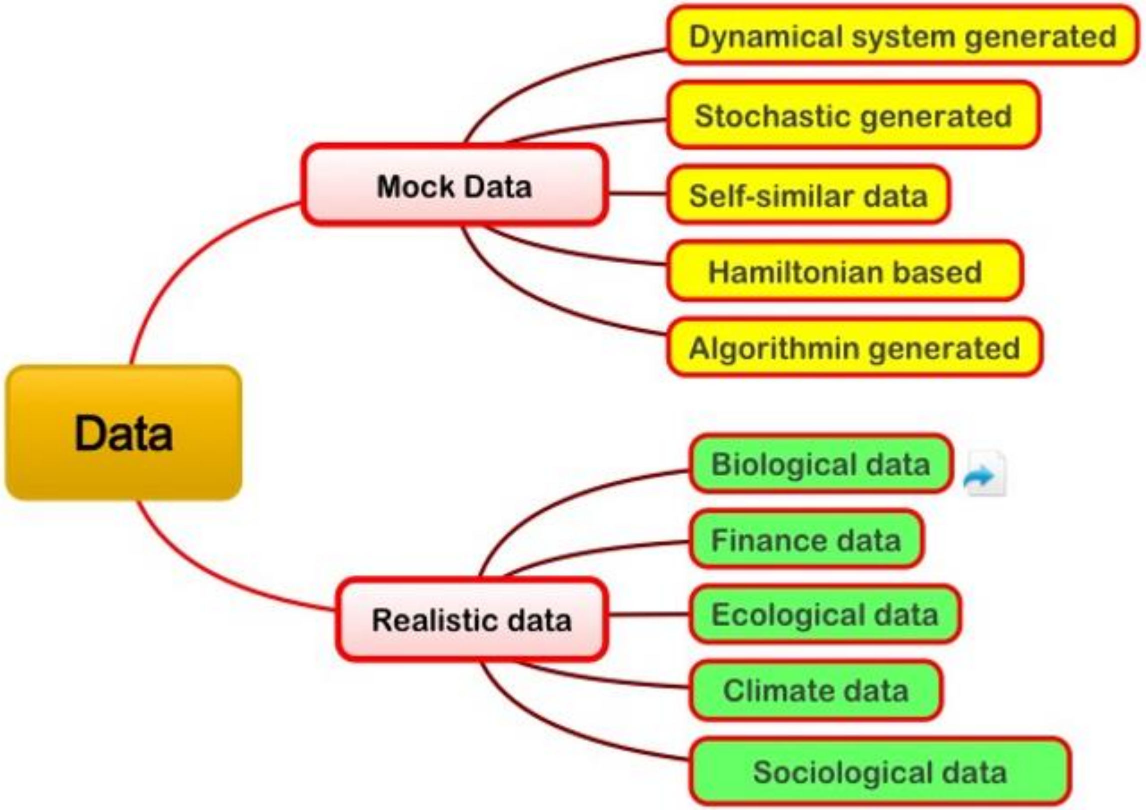
Rather than just moving towards a steady state, complex systems are often active and responding to the environment - the difference between a ball that rolls to the bottom of a hill and stops and a bird that adapts to wind currents while flying. This adaptation can happen at multiple scales: cognitive, through learning and psychological development; social, via sharing information through social ties; or even evolutionary, through genetic variation and natural selection. When the components are damaged or removed, these systems are often able to adapt and recover their previous functionality, and sometimes they become even better than before. This can be achieved by robustness, the ability to withstand perturbations; resilience, the ability to go back to the original state after a large perturbation; or adaptation, the ability to change the system itself to remain functional and survive. Complex systems with these properties are known as complex adaptive systems.



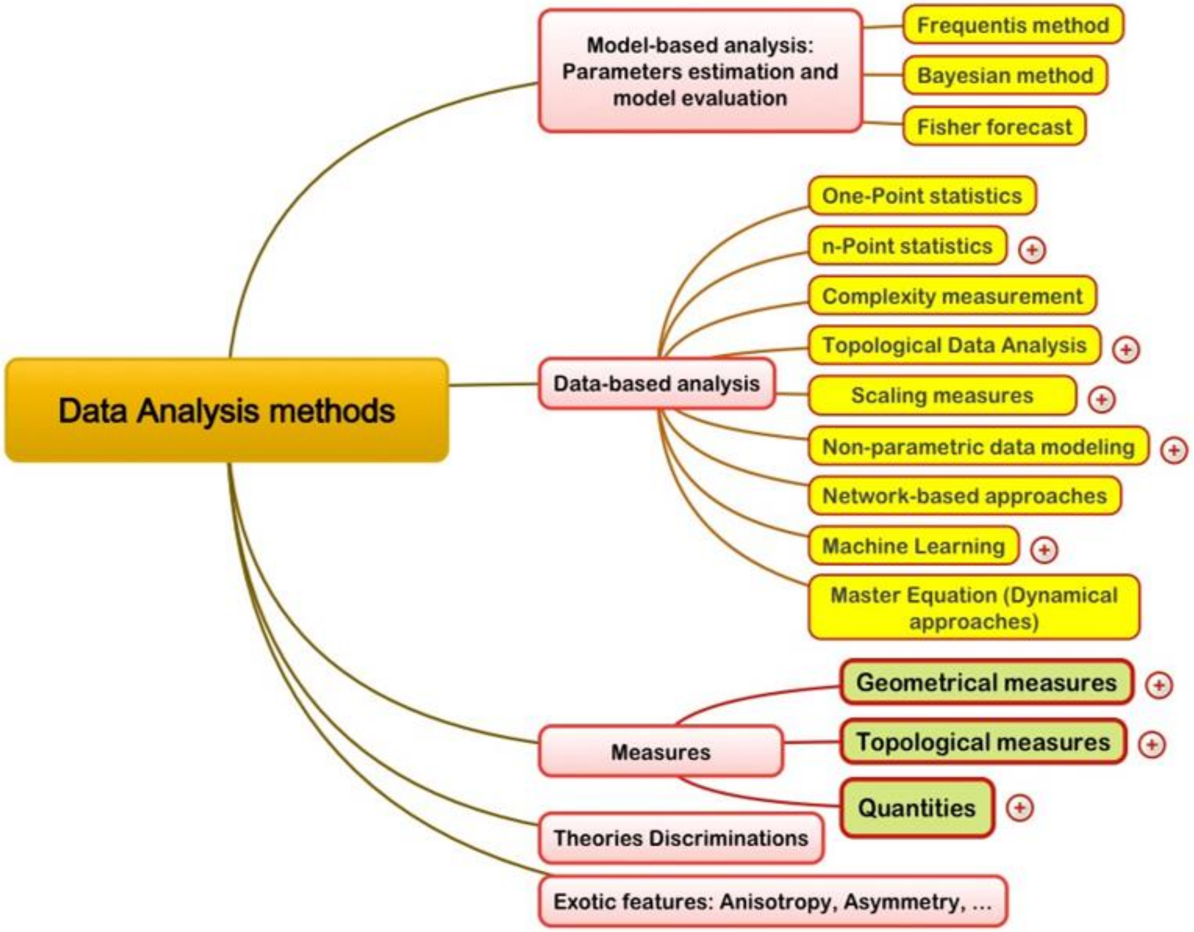
Critical Transition

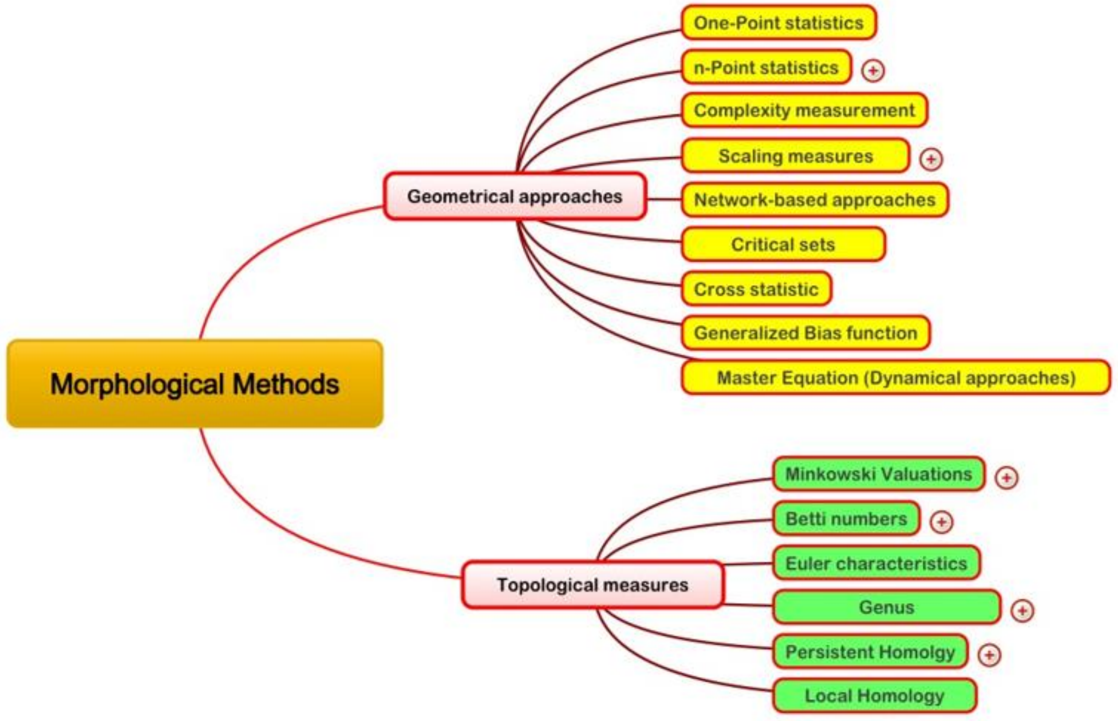


DATA TYPES

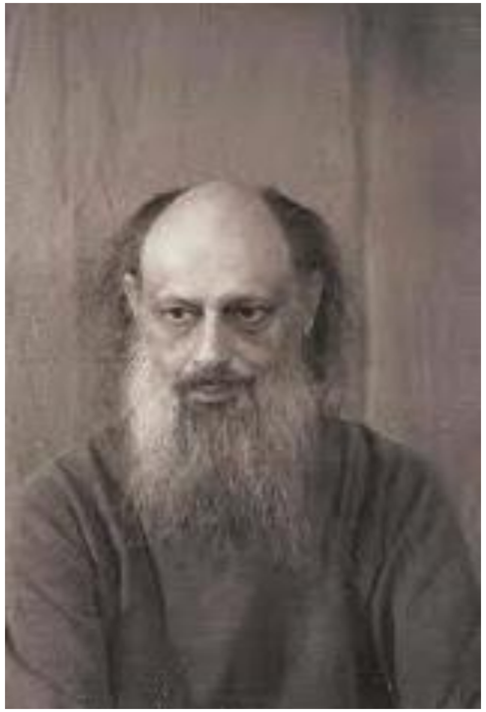

Data
Mock Data
Dynamical system generated
Stochastic generated
Self-similar data
Hamiltonian based
Algorithmin generated
Realistic data
Biological data
Finance data
Ecological data
Climate data
Sociological data

DATA ANALYSIS METHOD

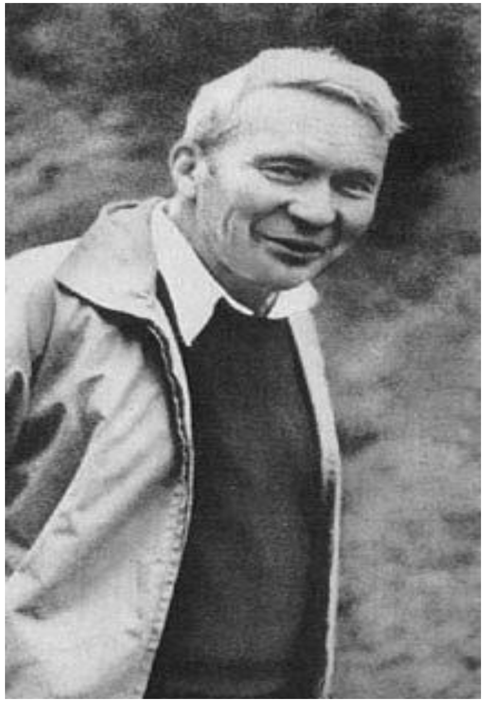




KOLMOGOROV COMPLEXITY



Solomonov
In many hypothesies that fits observed data which one should be preferred?



Kolmogrov
Correct Definition of randomness

Chaitin
Infromation should be relied on computation rather than probability

KOLMOGROV COMPLEXITY

$$C(x) = \min_{p}\{\text{length}(p) : U(p) = x\}$$

KOLMOGROV COMPLEXITY

$$C(x) = \min_p \{\text{length}(p) : U(p) = x\}$$

"write ab 16 times"	*"write 4c1j5b2p0cv4w1x8rx2y39umgw5q85s7"*
`abababababababababababababababab`	`4c1j5b2p0cv4w1x8rx2y39umgw5q85s7`

Lempel-Ziv Complexity

Lempel–Ziv complexity can be used to measure the repetitiveness of binary sequences and text, like song lyrics or prose. Fractal dimension estimates of real-world data have also been shown to correlate with Lempel–Ziv complexity.

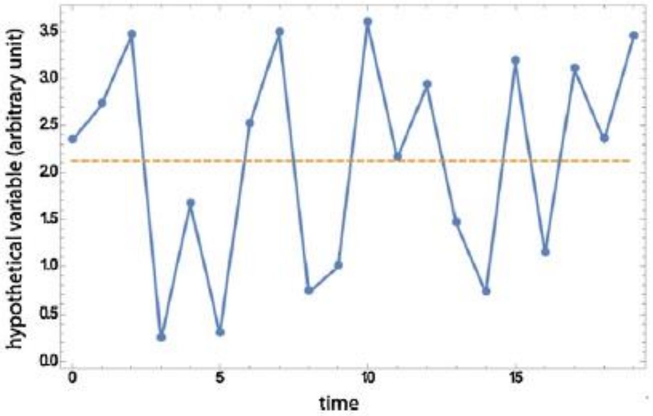
'100111011000010'

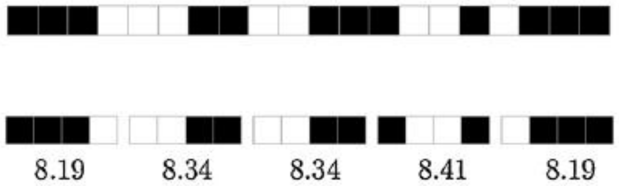
1 / 0 / 01 / 11 / 10 / 110 / 00 / 010

'101010101010101010'

1 / 0 / 10 / 101 / 01 / 010/ 1010

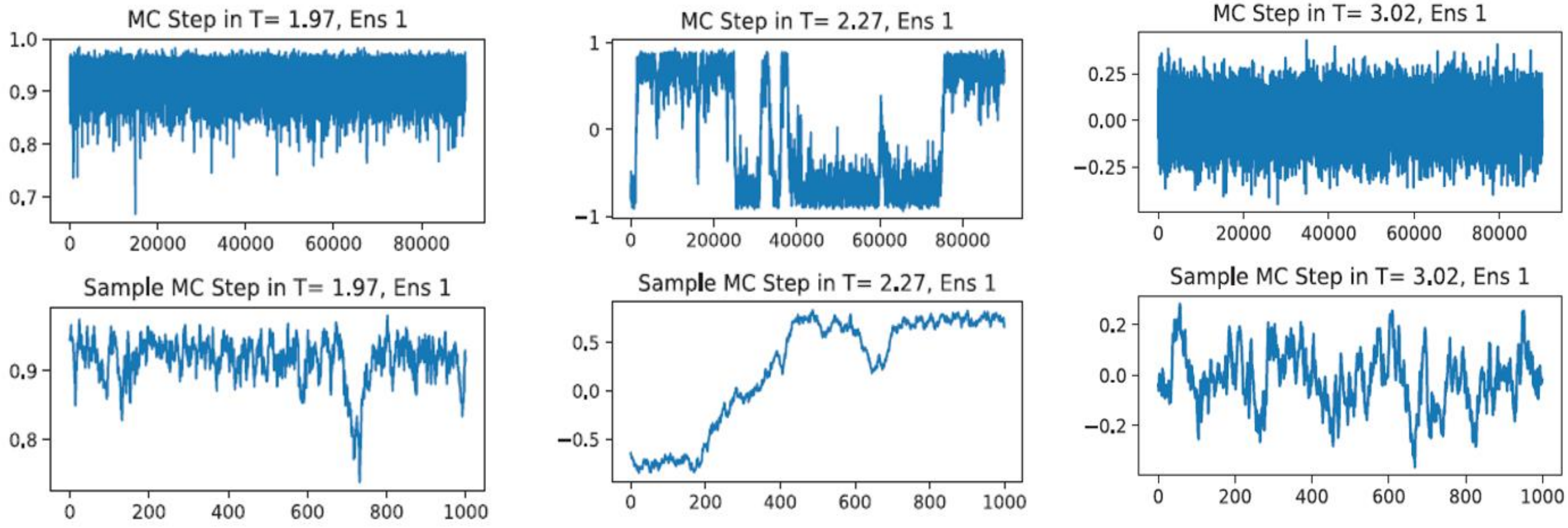
Block Decomposition Method





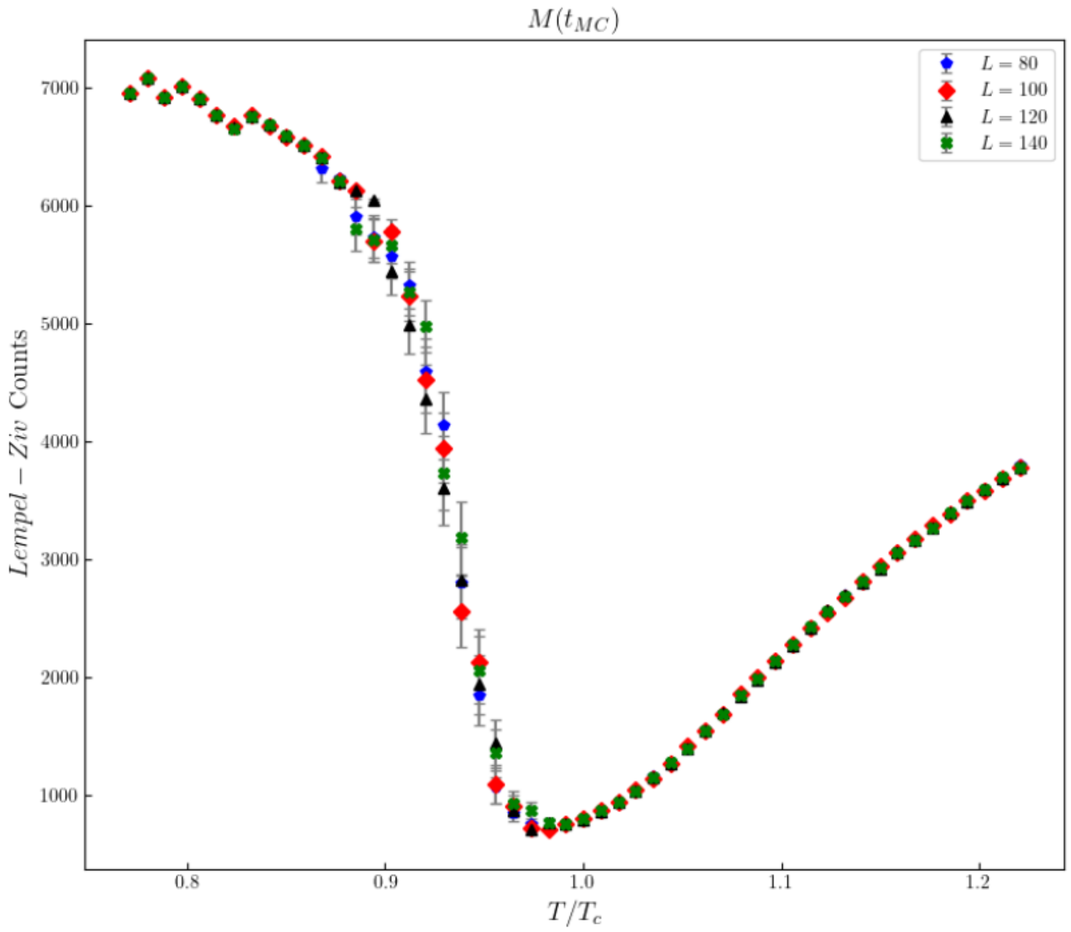
$$K_{BDM} = 8.19 + (\log_2 2 + 8.34) + 8.41 + 8.19 = 34.13$$

2D Ising Model



2D ISING MODEL

Lempel-Ziv Complexity Measure :
A new approach to the problem of evaluating the complexity ("randomness") of finite sequences is presented. The proposed complexity measure is related to the number of steps in a self-delimiting production process by which a given sequence is presumed to be generated. It is further related to the number of distinct substrings and the rate of their occurrence along the sequence. The derived properties of the proposed measure are discussed and motivated in conjunction with other well-established complexity criteria.



NEXT?

THANKS FOR YOUR ATTENTION