Footprint of Cosmic Strings in the Cosmic Microwave Background through the Conditional Moments of the First Derivative

4^{th} ISRD Meeting

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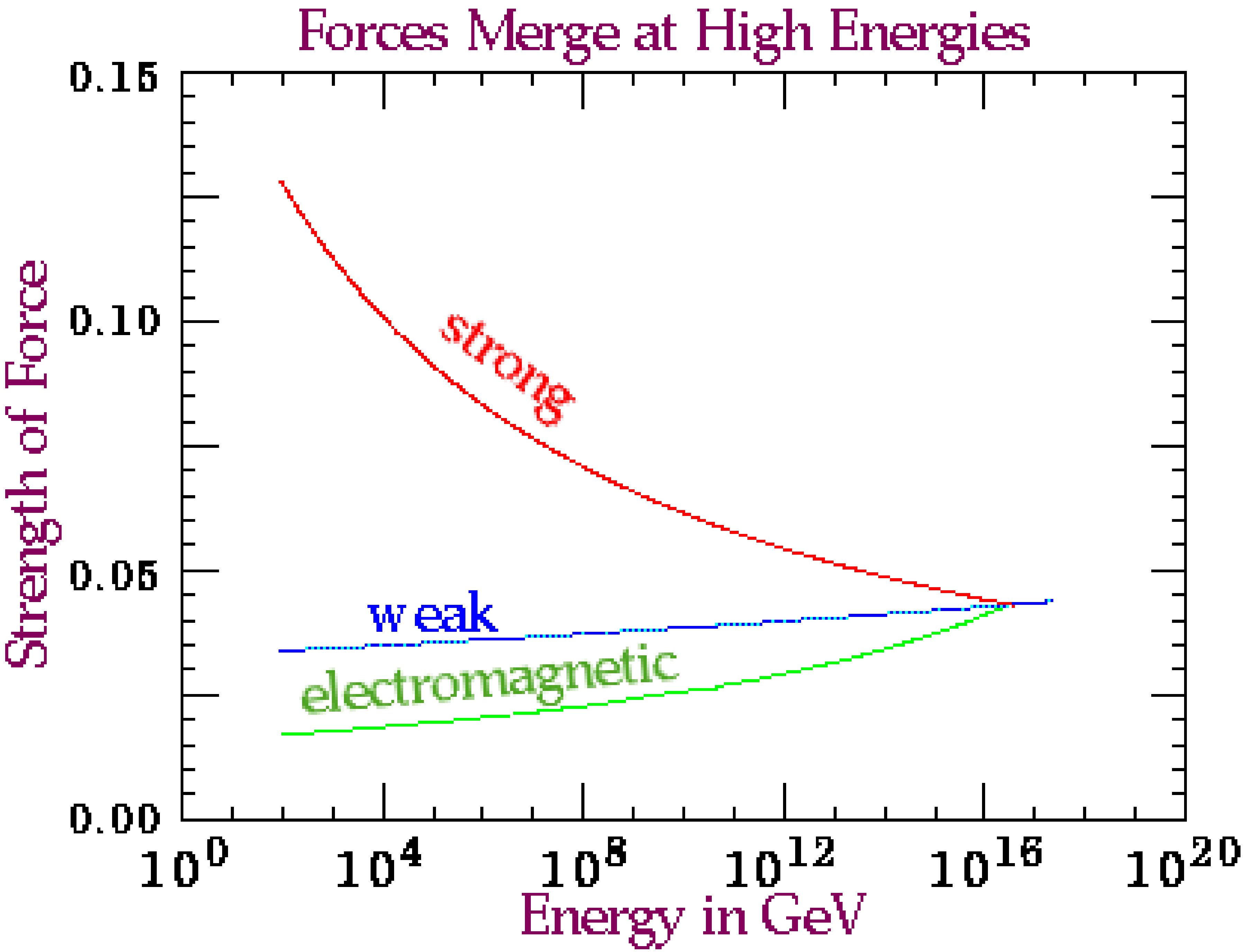
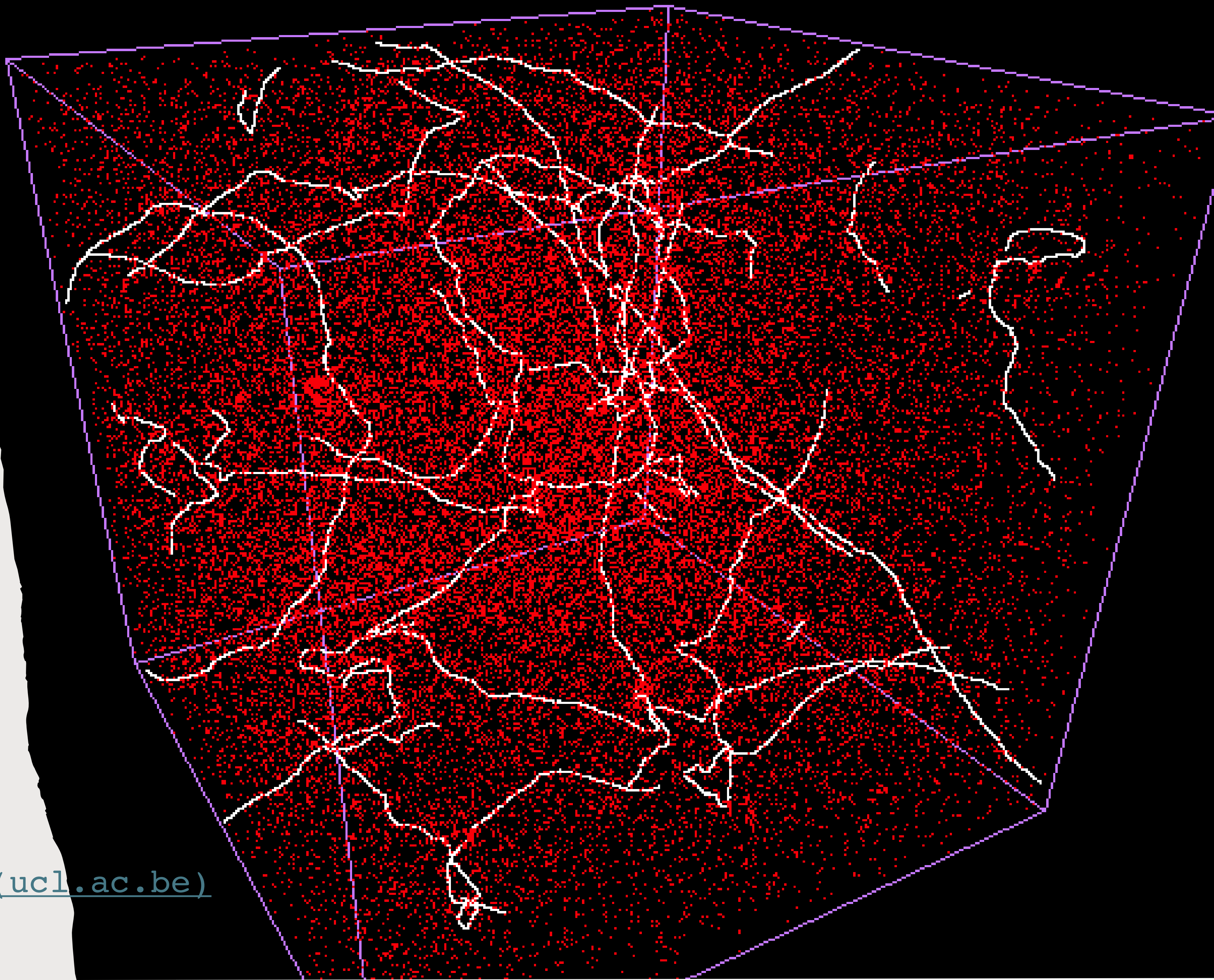
Dated: 02/July/2024

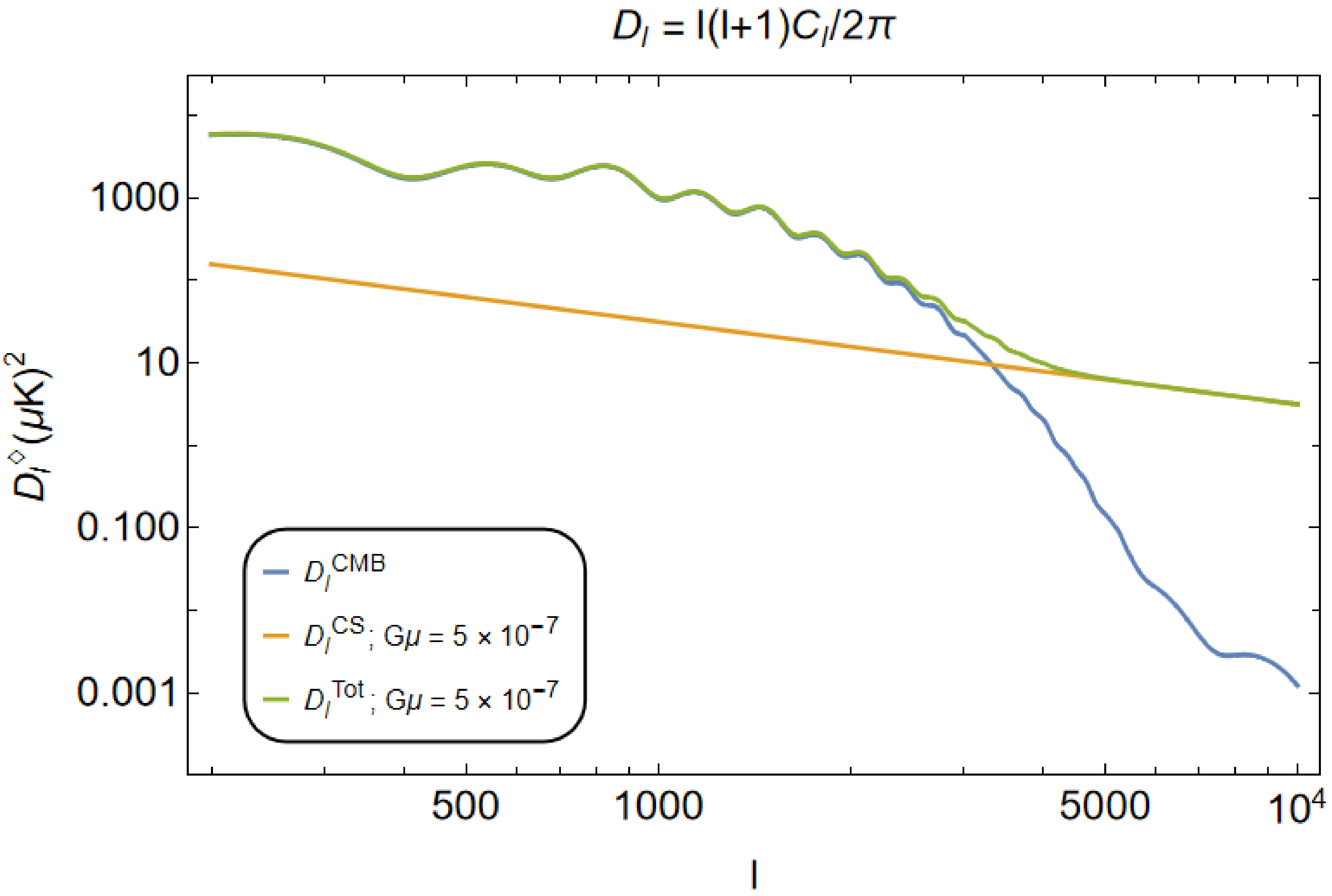

Forces Merge at High Energies
0.15
0.10
0.05
0.00
Strength of Force
strong
weak
electromagnetic
10^0
10^4
10^8
10^{12}
10^{16}
10^{20}
Energy in GeV

Cosmic Strings

Gμ

[Glance on cosmic strings (ucl.ac.be)]





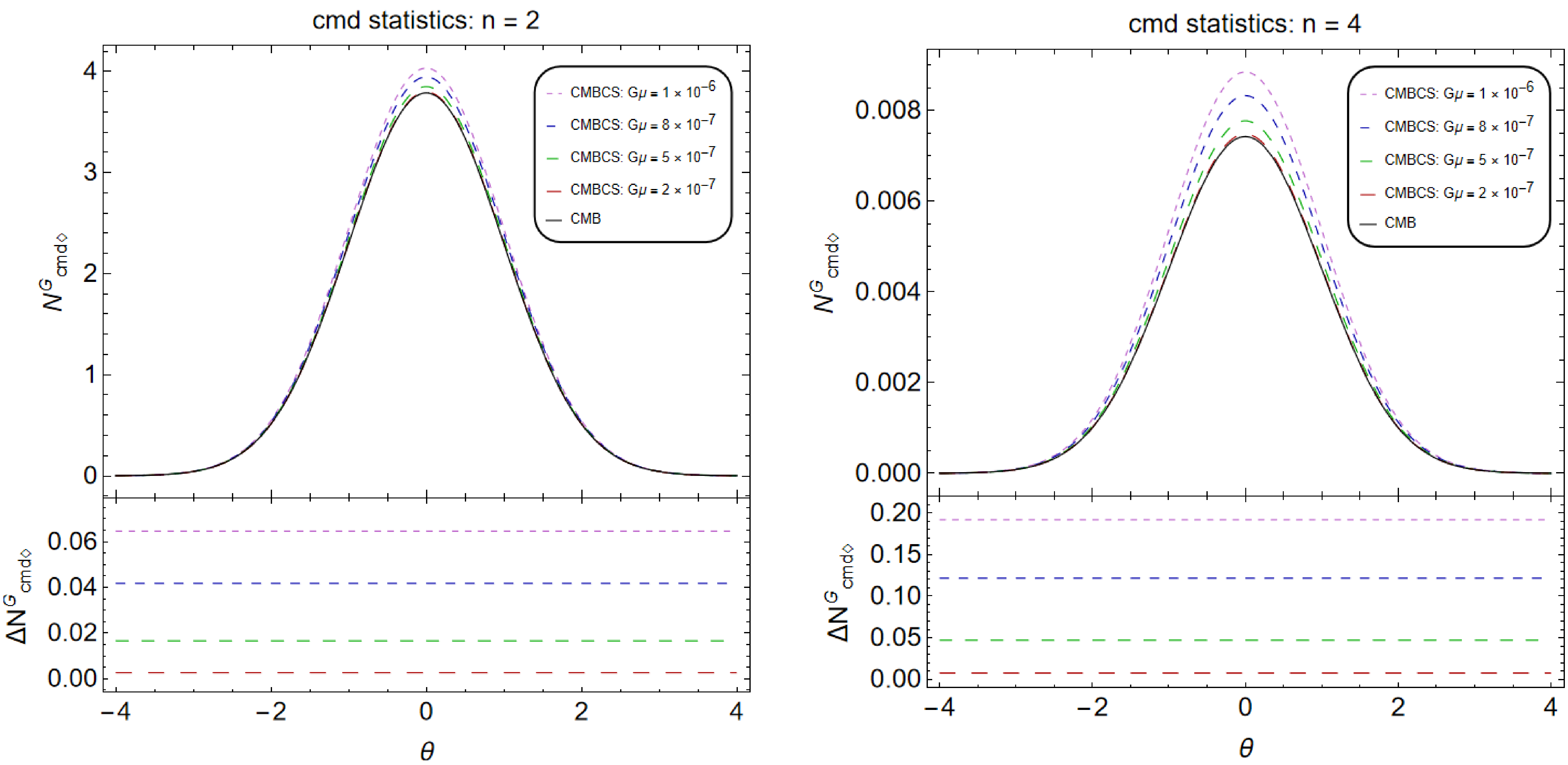
$D_l = l(l+1)C_l/2\pi$
1000
10
0.100
0.001
$D_l^{\diamond}(\mu K)^2$
500
1000
5000
10^4
l
D_l^{CMB}
D_l^{CS}; $G\mu = 5 \times 10^{-7}$
D_l^{Tot}; $G\mu = 5 \times 10^{-7}$

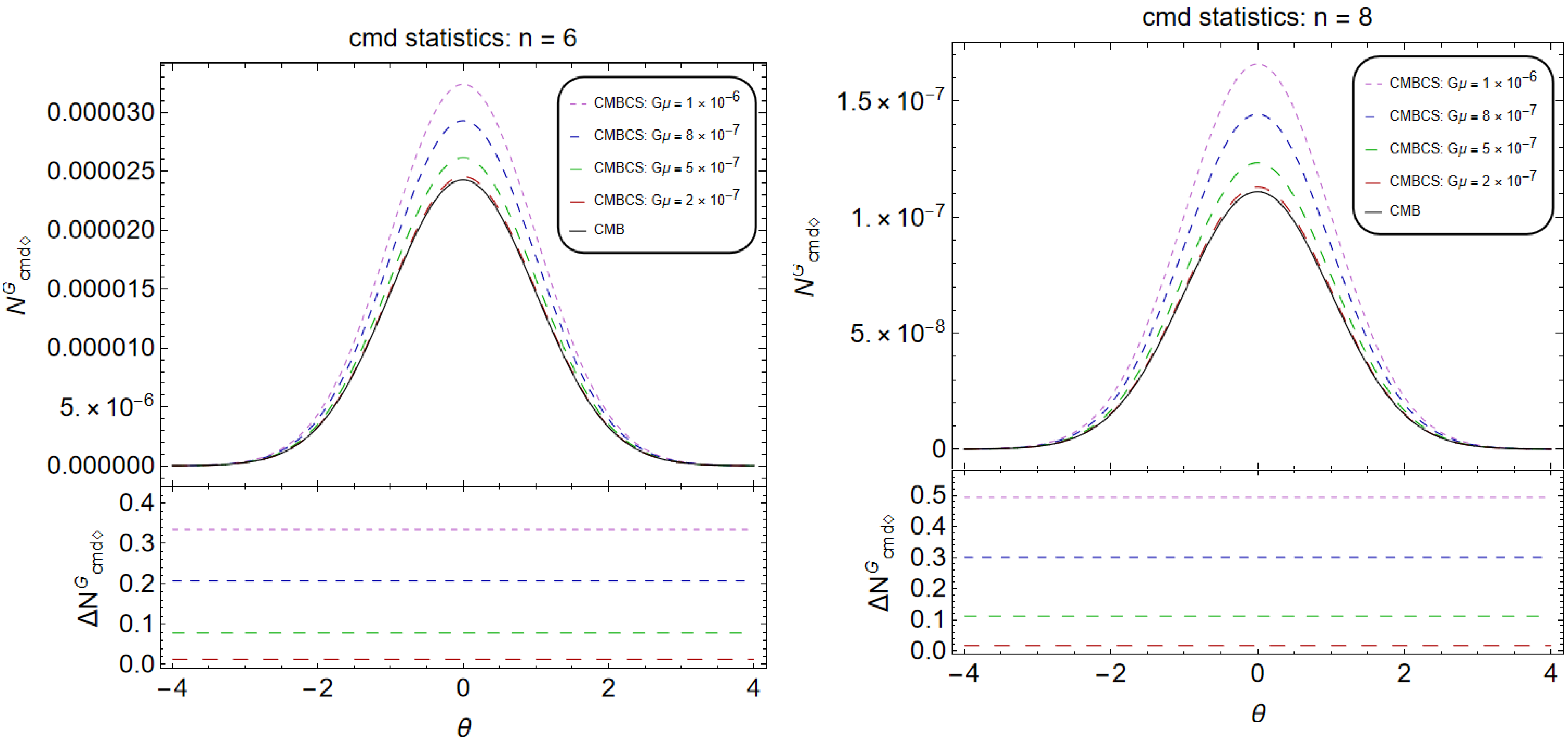
Morphology → Minkowski Functionals → Genus Statistics

Conditional Moments of the First Derivative

$$\mathcal{G} = \delta_D(\delta^{(r,s)} - \vartheta\sigma_0^{(r,s)})(\delta_{,i}^{(r,s)})^n$$

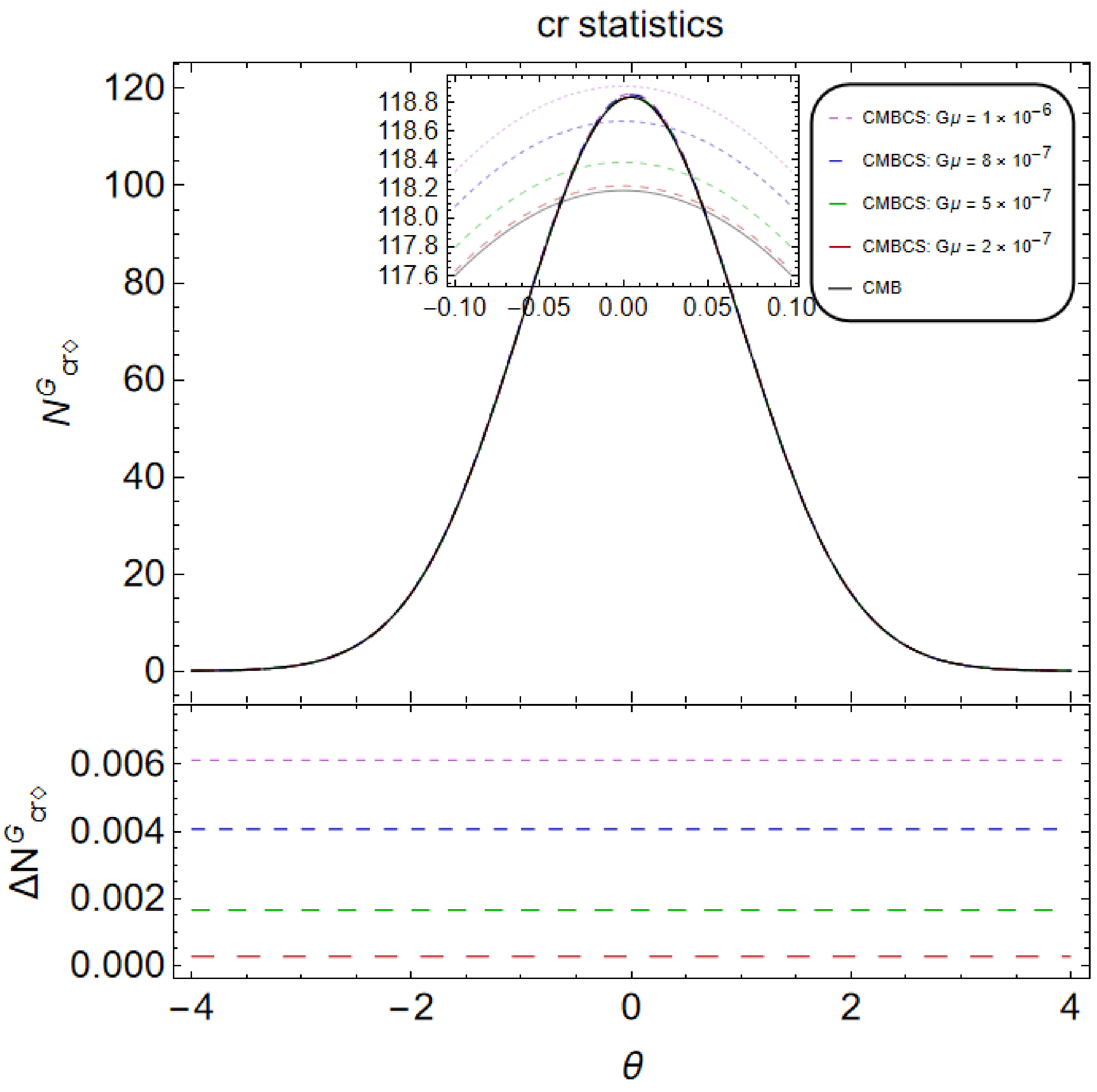
Gaussian Case

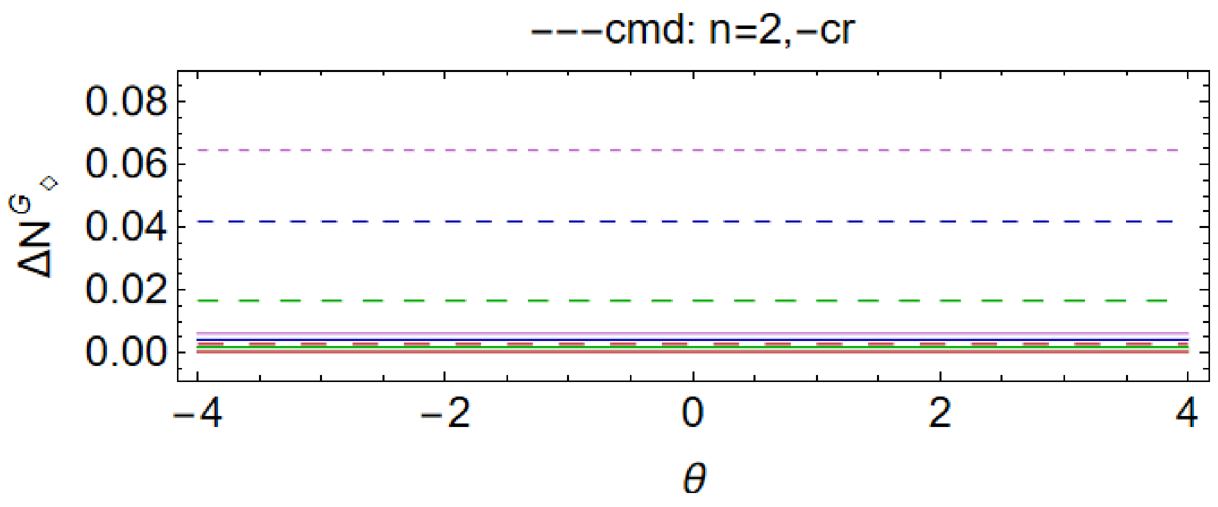

cmd statistics: n = 2
cmd statistics: n = 4
CMBCS: Gμ = 1 × 10^−6
CMBCS: Gμ = 8 × 10^−7
CMBCS: Gμ = 5 × 10^−7
CMBCS: Gμ = 2 × 10^−7
CMB
N^G_cmd◇
ΔN^G_cmd◇
θ

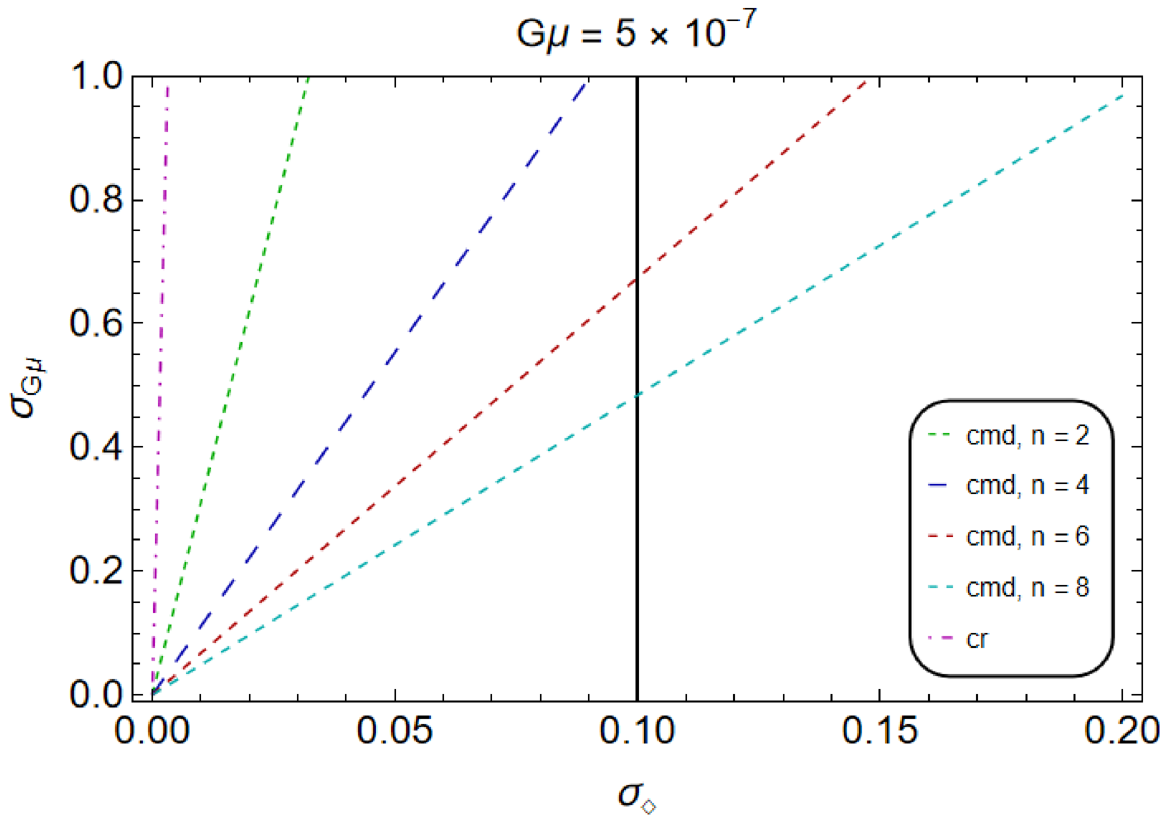


cmd statistics: n = 6
CMBCS: Gμ = 1 × 10^−6
CMBCS: Gμ = 8 × 10^−7
CMBCS: Gμ = 5 × 10^−7
CMBCS: Gμ = 2 × 10^−7
CMB
N^G_cmd◇
ΔN^G_cmd◇
θ
cmd statistics: n = 8

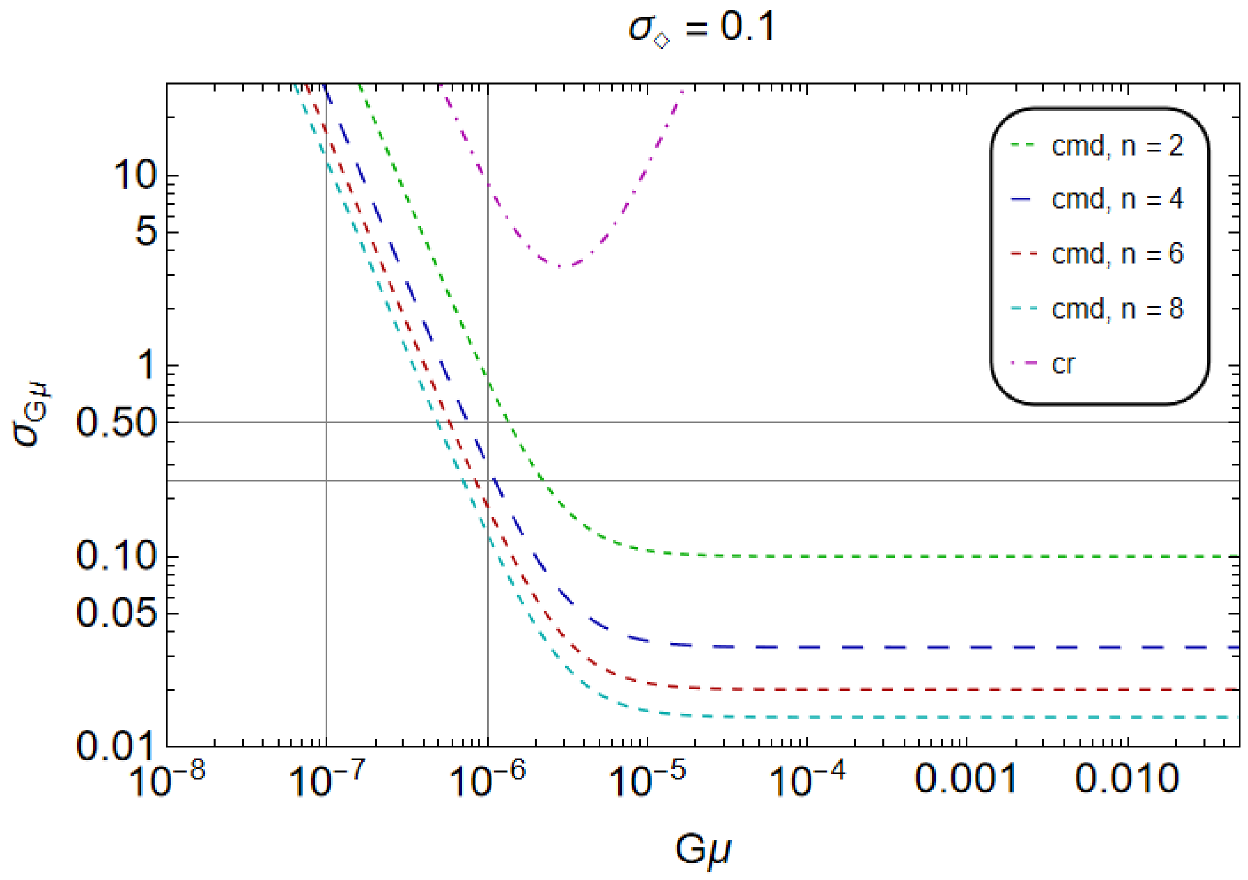
Crossing Statistics





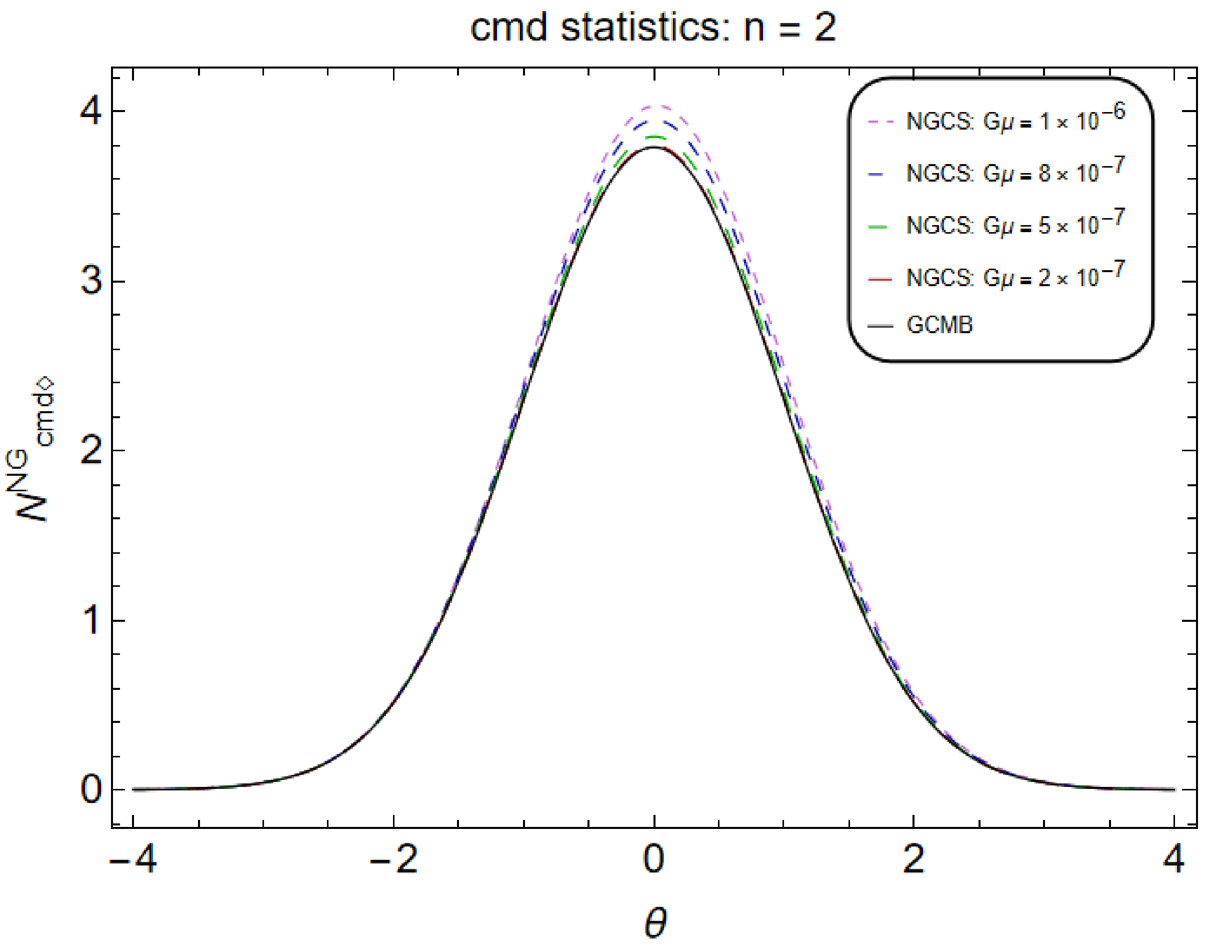


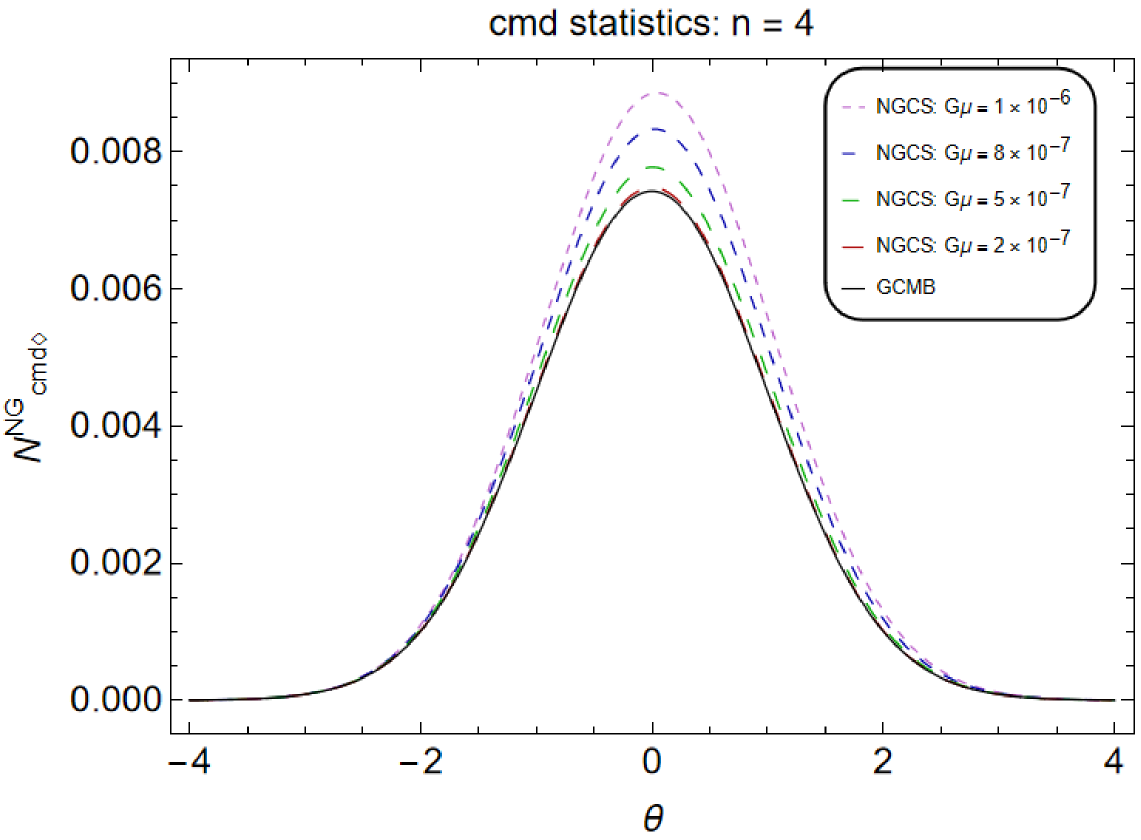
Statistical Error

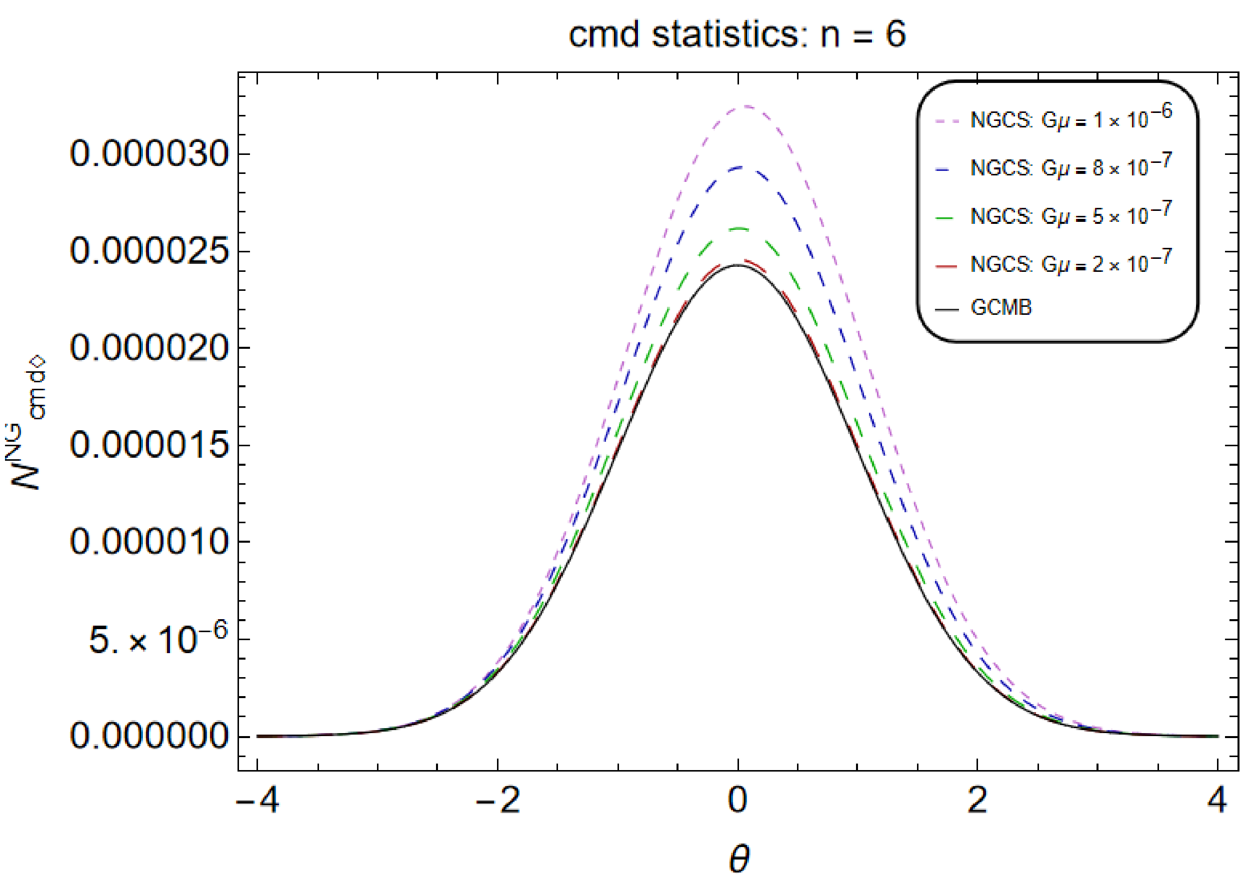


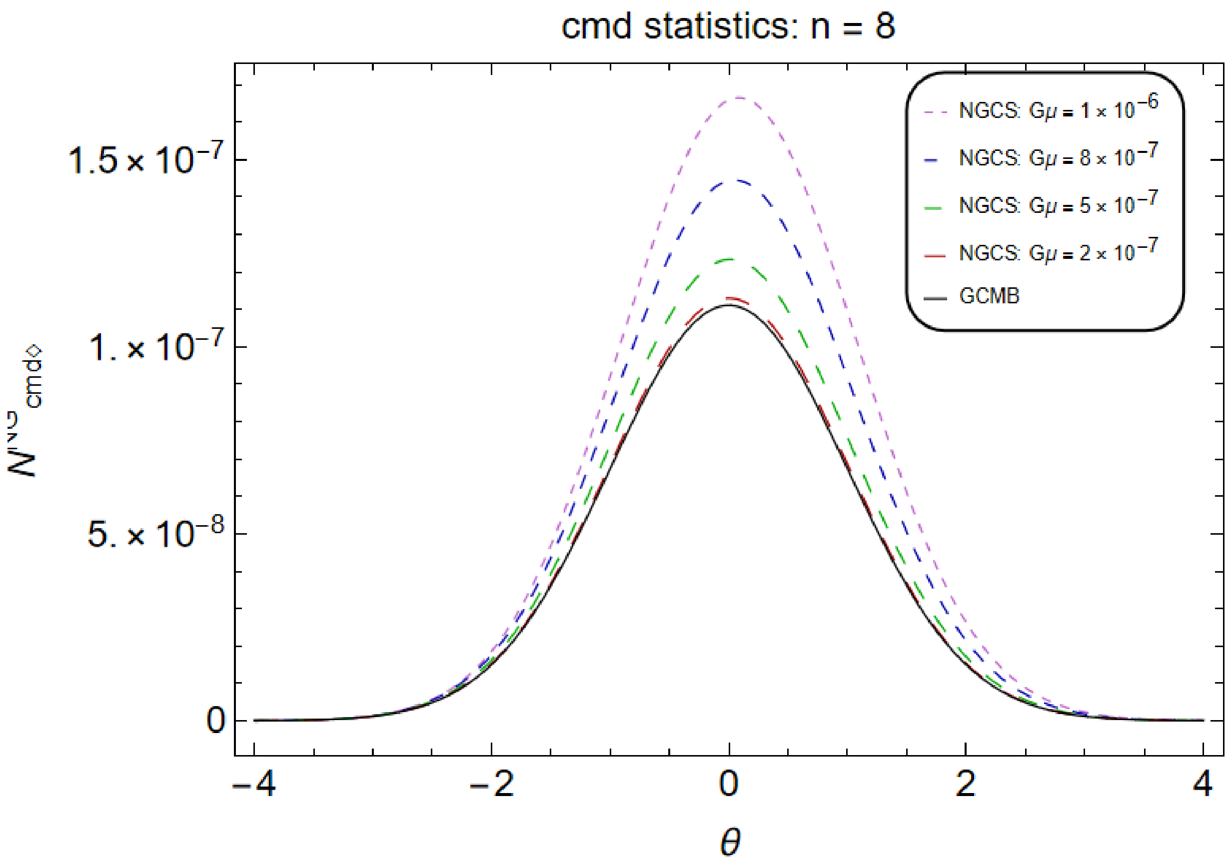
Non-Gaussianity

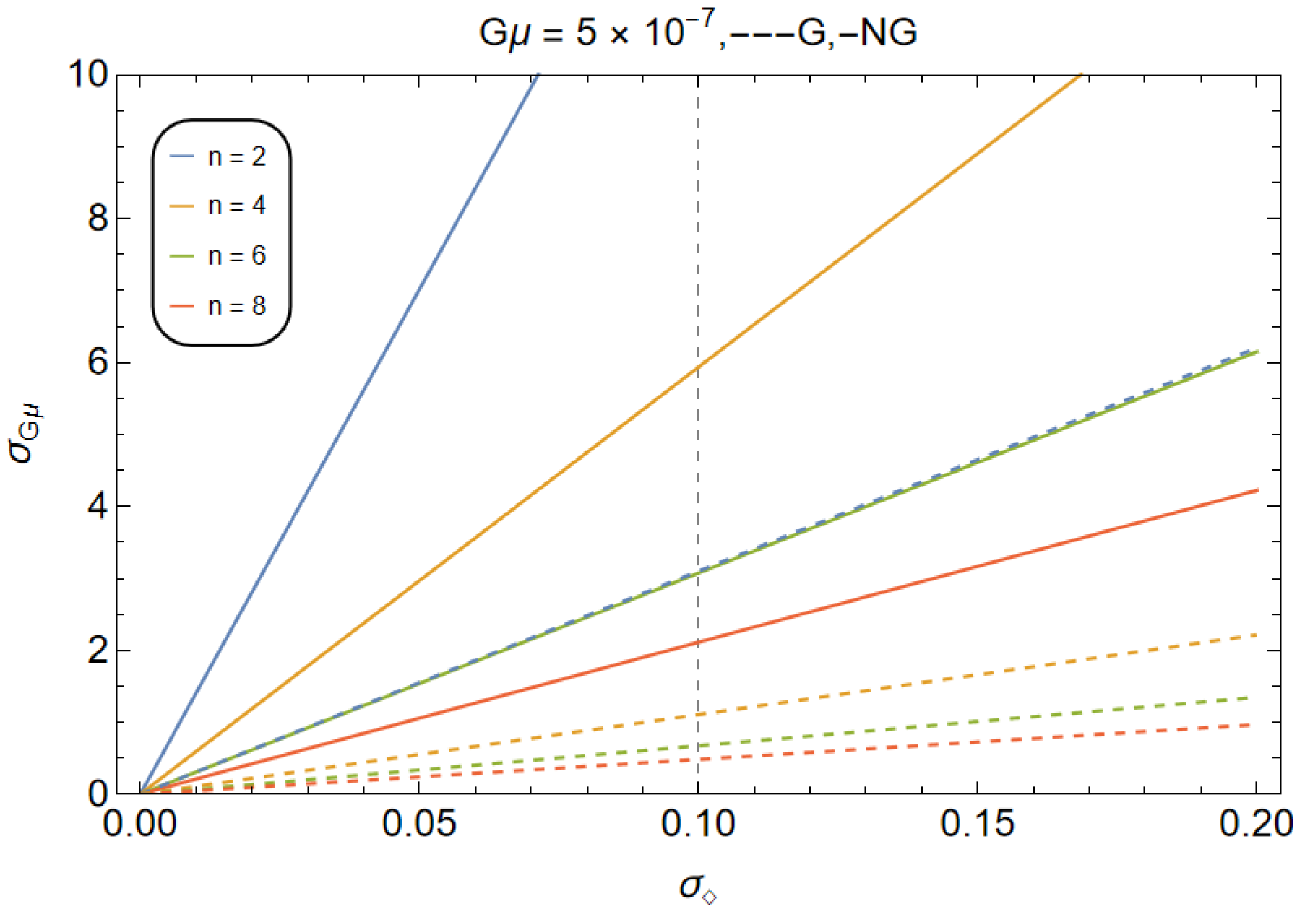
cmd Statistics






cmd statistics: n = 6
NGCS: Gμ = 1 × 10^−6
NGCS: Gμ = 8 × 10^−7
NGCS: Gμ = 5 × 10^−7
NGCS: Gμ = 2 × 10^−7
GCMB
0.000030
0.000025
0.000020
0.000015
0.000010
5. × 10^−6
0.000000
−4
−2
0
2
4
θ


cmd statistics: n = 8
NGCS: Gμ = 1 × 10^−6
NGCS: Gμ = 8 × 10^−7
NGCS: Gμ = 5 × 10^−7
NGCS: Gμ = 2 × 10^−7
GCMB
1.5 × 10^−7
1. × 10^−7
5. × 10^−8
0
−4
−2
0
2
4
θ



Gμ = 5 × 10^-7,---G,-NG
n = 2
n = 4
n = 6
n = 8
σ_Gμ
σ_◇
0.00
0.05
0.10
0.15
0.20
0
2
4
6
8
10

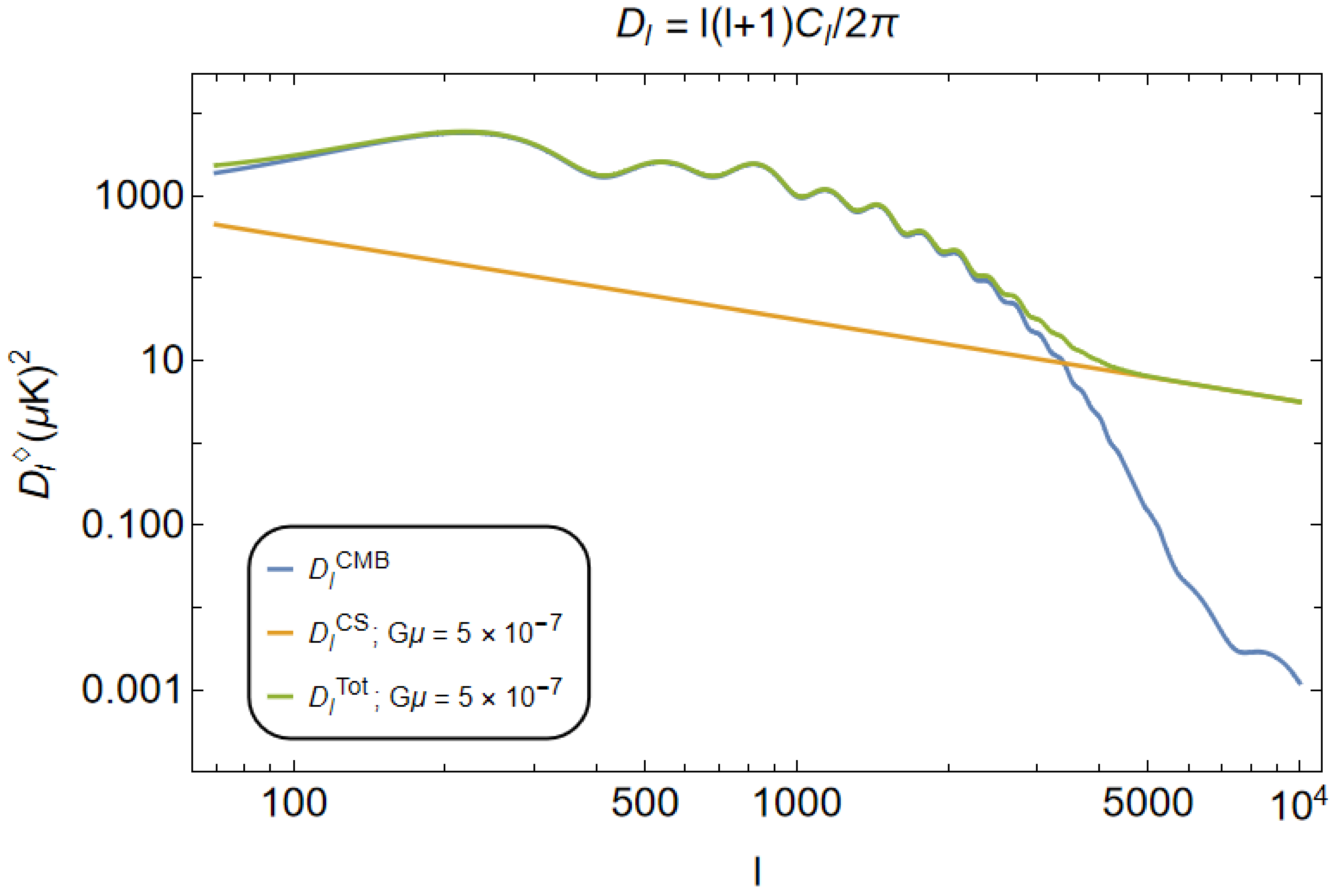
RESTRICTED AREA
CONSTRUCTION WORK IN PROGRESS

Future Tasks:

- Comparison with the observational data
- Calculate the p-value to constrain the parameter space of $G\mu$
- Test the robustness of our pipeline by introducing some noise and beam effects

Thank you for your attention

Backup Slides



$D_l = l(l+1)C_l/2\pi$
$D_l^{\diamond}(\mu K)^2$
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D_l^{CS}; $G\mu = 5 \times 10^{-7}$
D_l^{Tot}; $G\mu = 5 \times 10^{-7}$

It follows from $\langle f \rangle = 0$ that $\langle A_\mu \rangle = 0$, and the first several cumulants are given by

$$M^{(1)}_{\mu} = 0 \ ,$$
$$M^{(2)}_{\mu_1\mu_2} = \langle A_{\mu_1} A_{\mu_2} \rangle \ ,$$
$$M^{(3)}_{\mu_1\mu_2\mu_3} = \langle A_{\mu_1} A_{\mu_2} A_{\mu_3} \rangle \ ,$$
$$M^{(4)}_{\mu_1\mu_2\mu_3\mu_4} = \langle A_{\mu_1} A_{\mu_2} A_{\mu_3} A_{\mu_4} \rangle - \langle A_{\mu_1} A_{\mu_2} \rangle \langle A_{\mu_3} A_{\mu_4} \rangle - \langle A_{\mu_1} A_{\mu_3} \rangle \langle A_{\mu_2} A_{\mu_4} \rangle - \langle A_{\mu_1} A_{\mu_4} \rangle \langle A_{\mu_2} A_{\mu_3} \rangle \ ,$$

$$\langle F \rangle = \langle F \rangle_{\mathrm{G}} + \frac{1}{3!} \sum \hat{M}^{(3)}_{\mu_1\mu_2\mu_3} \langle F_{,\mu_1\mu_2\mu_3} \rangle_{\mathrm{G}} \sigma_0 + \left[\frac{1}{4!} \sum \hat{M}^{(4)}_{\mu_1\mu_2\mu_3\mu_4} \langle F_{,\mu_1\mu_2\mu_3\mu_4} \rangle_{\mathrm{G}} + \frac{1}{2(3!)^2} \sum \hat{M}^{(3)}_{\mu_1\mu_2\mu_3} \hat{M}^{(3)}_{\nu_1\nu_2\nu_3} \langle F_{,\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3} \rangle_{\mathrm{G}} \right] \sigma_0^2 + \mathcal{O}(\sigma_0^3)$$

$$\Theta_{\mathrm{cmd}} = \frac{N_{\mathrm{cmd}}{}^{\mathrm{CMBCS}}}{N_{\mathrm{cmd}}{}^{\mathrm{CMB}}}$$

$$\sigma_{G\mu}{}^2 = \left(\frac{\partial \mathrm{In}\ (\Theta_{\mathrm{cmd}})}{\partial \mathrm{In}\ (G\mu)} \right)^{-2} \sigma_{\diamond}{}^2$$