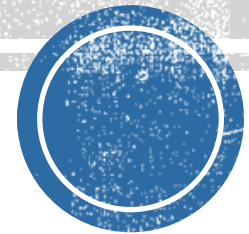


# GW to constrain alternative dark energy models



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**SUPERVISOR : Prof. S.M.S MOVAHEAD**  
**ISRD4**

# MY WORK

Understanding  
basics of GWs

1

How can we  
use it as a tool  
?!

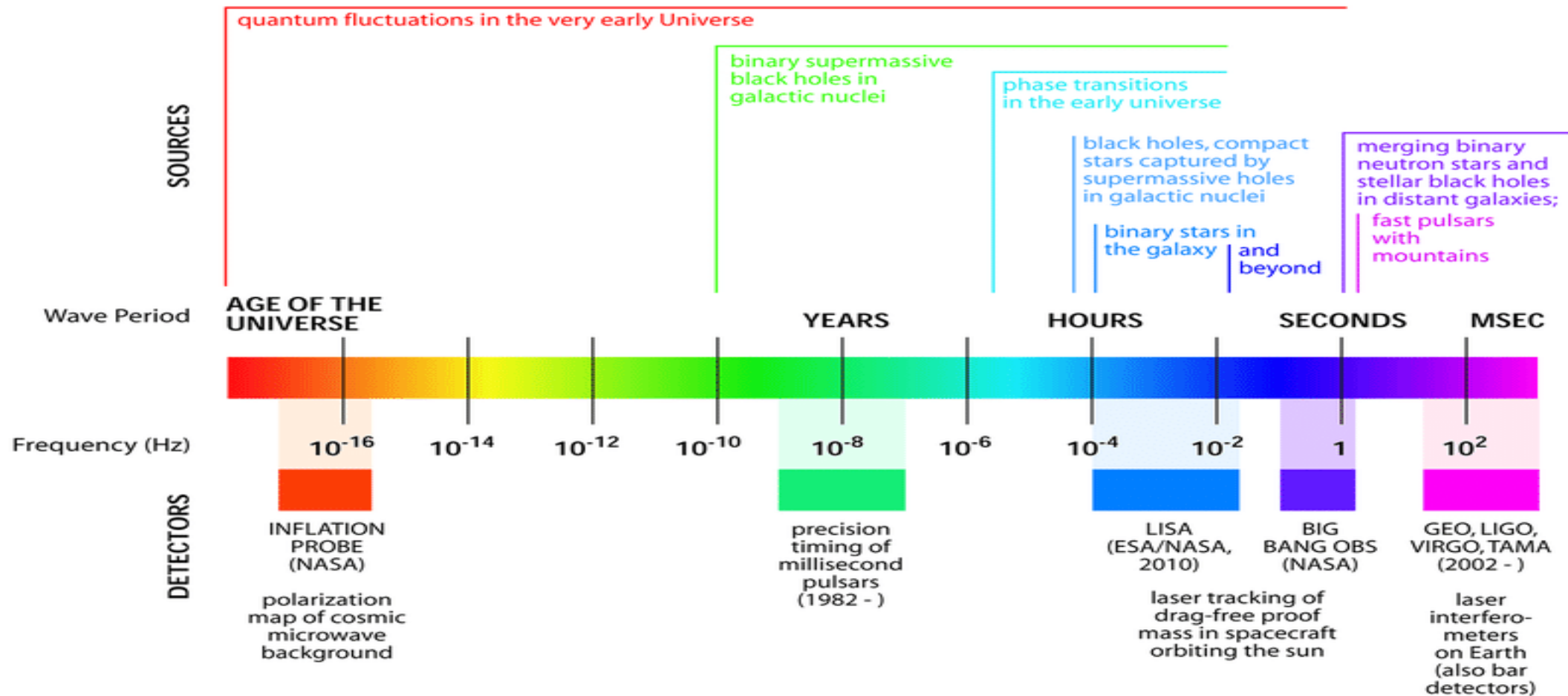
2

Simulating  
(coding)

3

# GWs SPECTRUM

## THE GRAVITATIONAL WAVE SPECTRUM



1

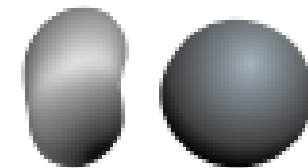
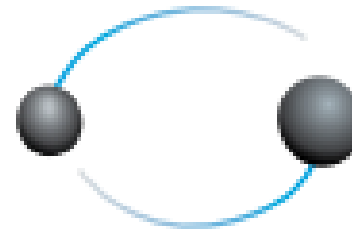
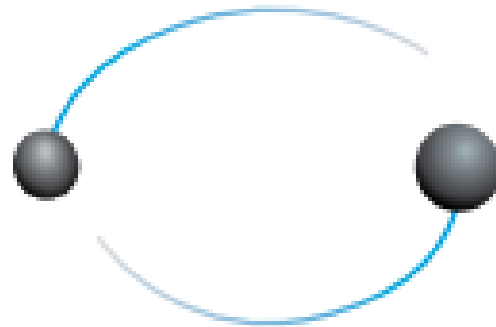
# Which sources?

BHNS

NSNS

Inspiral

Merger Ringdown



What is usage of GWs ?



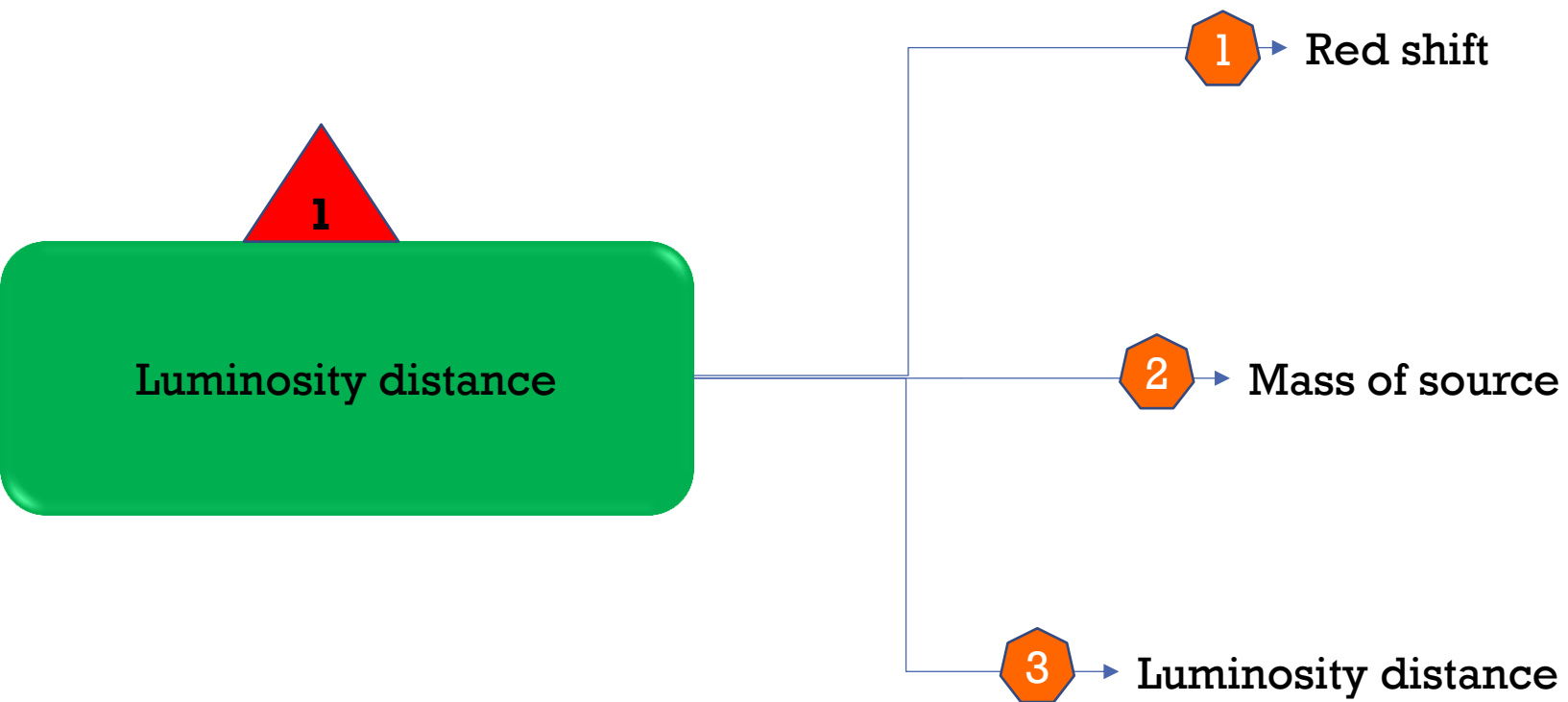
We can use it to put constrain on cosmological parameters

**1** Luminosity distance

**2** GWs weak lensing

**3** GWs strong lensing

**4** Distance sum rule



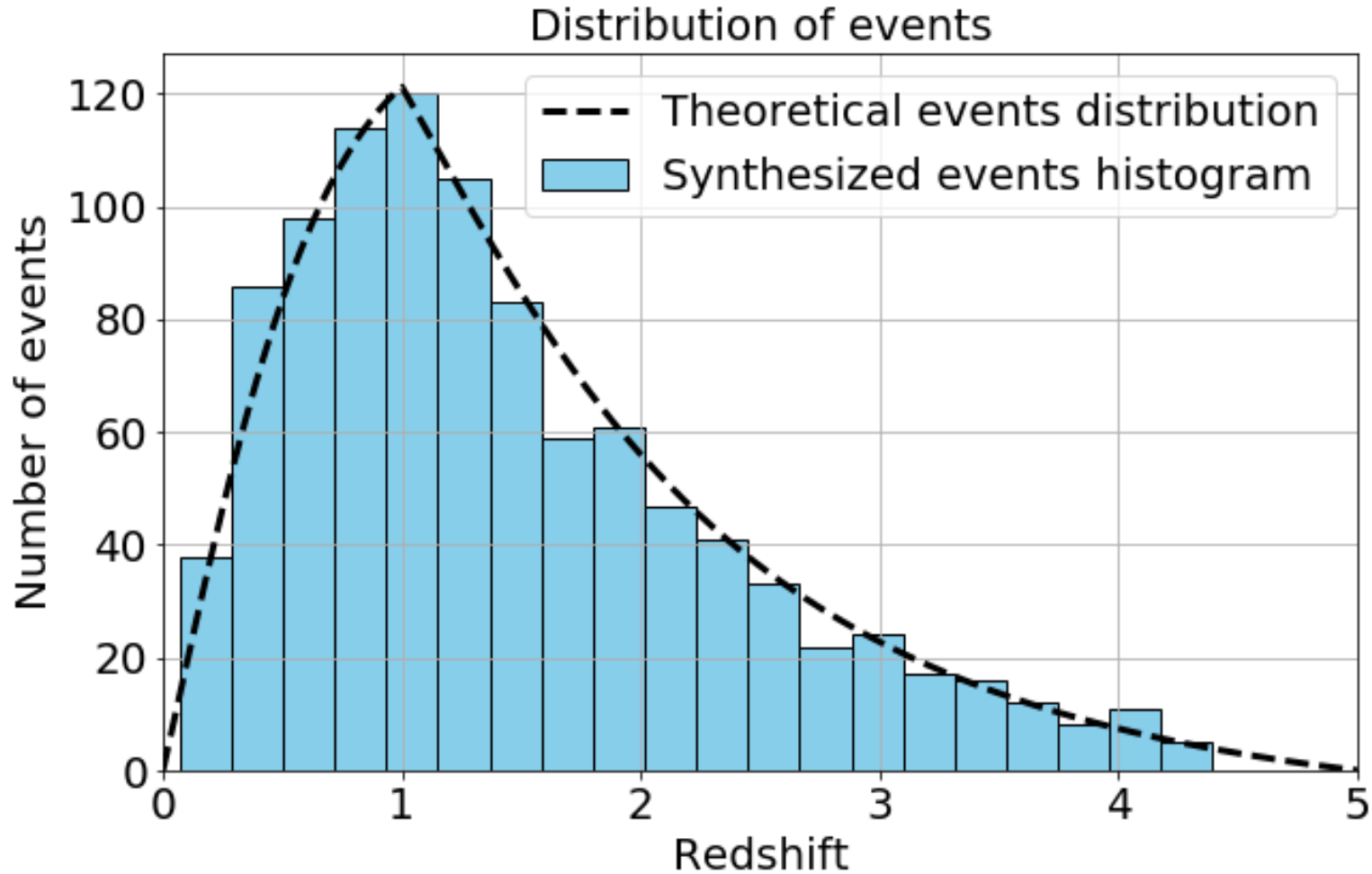
## Red shift

$$P(z) \propto \frac{4\pi d_c^2(z) R(z)}{H(z)(1+z)}$$

→ where  $d_c(z)$  represents the comoving distance at the redshift  $z$

→  $R(z)$  is the merger rate of binary system (BHNS or BNS)

$$R(z) = \begin{cases} 1 + 2z, & z \leq 1, \\ \frac{3}{4}(5 - z), & 1 < z < 5, \\ 0, & z \geq 5, \end{cases}$$



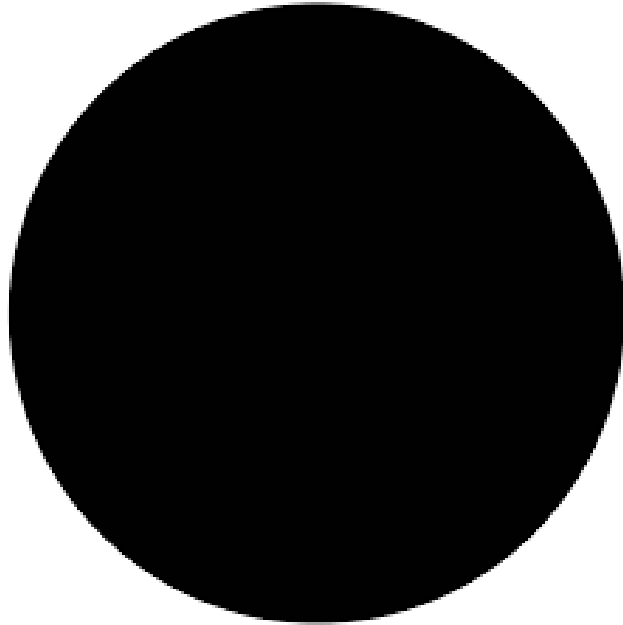
## DISTRIBUTION OF EVENTS

$$P(z) \propto \frac{4\pi d_C^2(z)R(z)}{H(z)(1+z)}$$

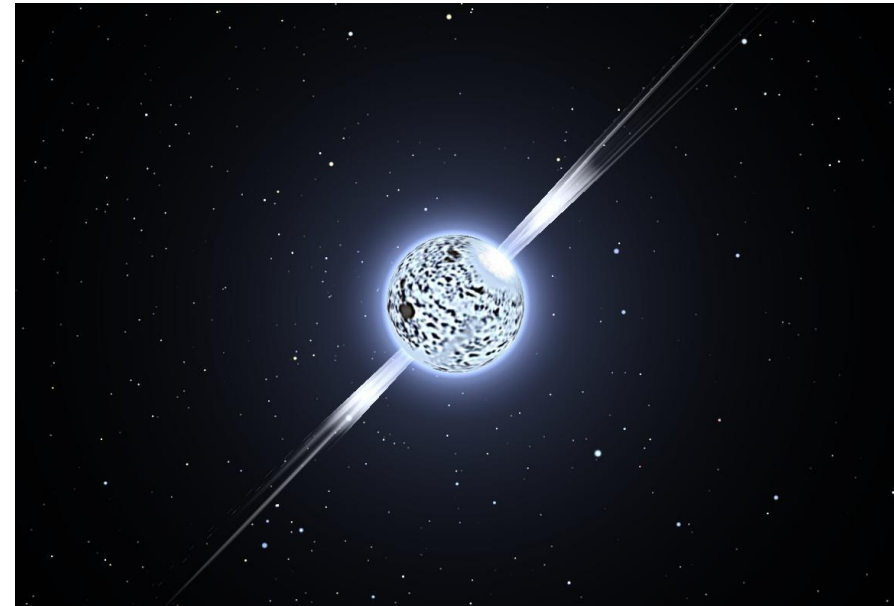
$$R(z) = \begin{cases} 1 + 2z, & z \leq 1, \\ \frac{3}{4}(5 - z), & 1 < z < 5, \\ 0, & z \geq 5, \end{cases} \quad [3]$$



Mass of sources



$[3-10] M_{\odot}$



$[1-2] M_{\odot}$

Luminosity distance  
&  
GW

$\psi$  is the polarization angle,  
and  $(\theta, \varphi)$  are angles  
describing the location of  
the source in the sky,  
relative to the detector .

$$h(t) = F_+(\theta, \varphi, \psi)h_+(t) + F_\times(\theta, \varphi, \psi)h_\times(t)$$

Beam pattern functions

$$F_+^{(1)}(\theta, \varphi, \psi) = \frac{\sqrt{3}}{2} \begin{bmatrix} \frac{1}{2} (1 + \cos^2 \theta) \cos(2\varphi) \cos(2\psi) \\ -\cos(\theta) \sin(2\varphi) \sin(2\psi) \end{bmatrix}$$

$$F_\times^{(1)}(\theta, \varphi, \psi) = \frac{\sqrt{3}}{2} \begin{bmatrix} \frac{1}{2} (1 + \cos^2 \theta) \cos(2\varphi) \sin(2\psi) \\ +\cos(\theta) \sin(2\varphi) \cos(2\psi) \end{bmatrix}$$

Fourier space

$$H(f) = Af^{-7/6} \exp\left[i\left(2\pi ft_0 - \frac{\pi}{4} + 2\psi\left(\frac{f}{2}\right) - \varphi_{(2,0)}\right)\right]$$

$$A = \frac{1}{d_L} \sqrt{F_+^2(1 + \cos^2(\iota))^2 + F_\times^2 \cos^2(\iota)} \\ \times \sqrt{5\pi/96} \pi^{-7/6} M_c^{5/6}$$

[3-10]  $M_\odot$

**Black hole**

$$M_c = M\eta^{3/5}$$

$$M = m_1 + m_2$$

$$\eta = m_1 m_2 / M^2$$

[1-2]  $M_\odot$

**Neutron star**

# Luminosity distance

$$d_L = \begin{cases} \frac{1+z}{H_0 \sqrt{\Omega_k}} \sinh \left( \Omega_k \int_0^z \frac{d\tilde{z}}{E(\tilde{z})} \right) & \Omega_k > 0 \\ \frac{1+z}{H_0} \int_0^z \frac{d\tilde{z}}{E(\tilde{z})} & \Omega_k = 0 \\ \frac{1+z}{H_0 \sqrt{|\Omega_k|}} \sin \left( \sqrt{|\Omega_k|} \int_0^z \frac{d\tilde{z}}{E(\tilde{z})} \right) & \Omega_k < 0 \end{cases} \quad E(z) = H(z)/H_0$$

$$d_L(z) = (1+z) \int_0^z \frac{d\tilde{z}}{E(\tilde{z})}$$

$$H(z)^2 = H_0^2 \left\{ (1 - \Omega_m - \Omega_k) \exp \left[ 3 \int_0^z \frac{1 + \omega(\tilde{z})}{1 + \tilde{z}} d\tilde{z} \right] + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 \right\}$$

## Where is the usage of GWs ?

$$\sigma_{d_L} = \sqrt{(\sigma_{d_L}^{\text{inst}})^2 + (\sigma_{d_L}^{\text{lens}})^2}$$

$$\sigma_{d_L}^{\text{inst}} \simeq \frac{2d_L}{\rho}$$

$$\sigma_{d_L}^{\text{lens}} / d_L = 0.05z.$$

$$\rho^{(i)} = \sqrt{\langle \mathcal{H}^{(i)}, \mathcal{H}^{(i)} \rangle};$$

$$\langle a, b \rangle = 4 \int_{f_{\text{lower}}}^{f_{\text{upper}}} \frac{\tilde{a}(f)\tilde{b}^*(f) + \tilde{a}^*(f)\tilde{b}(f)}{2} \frac{df}{S_h(f)},$$

$$H(f) = A f^{-7/6} \exp\left[i\left(2\pi f t_0 - \frac{\pi}{4} + 2\psi\left(\frac{f}{2}\right) - \varphi_{(2,0)}\right)\right]$$

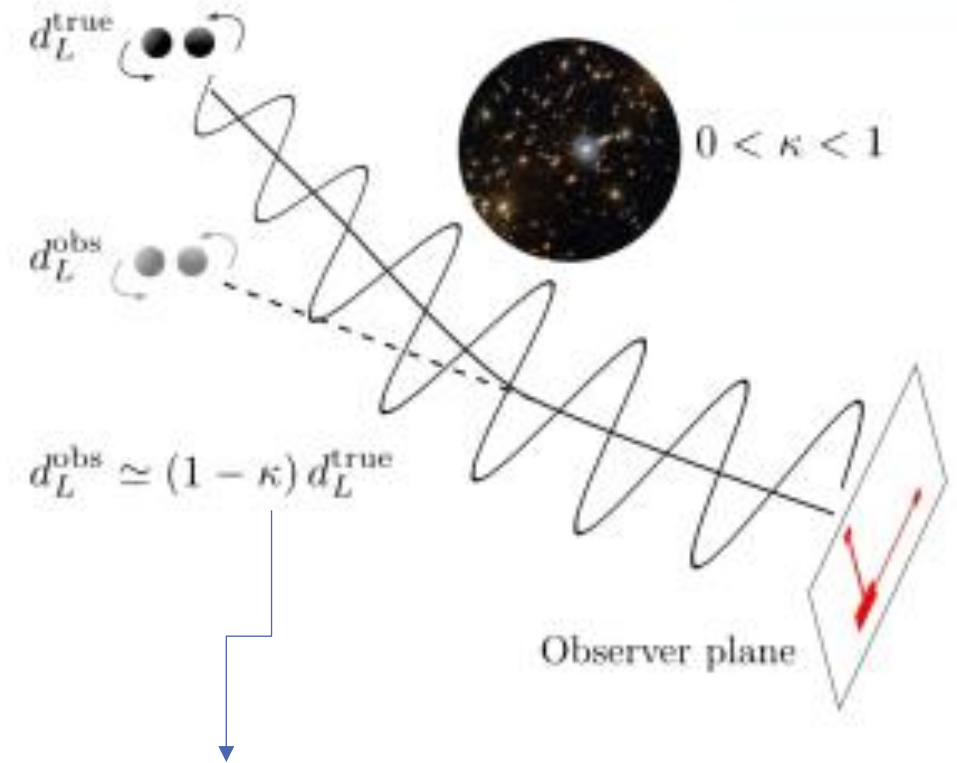
## Weak lensing

$$d_L^{\text{obs}} = \frac{1}{\sqrt{\mu}} d_L^{\text{true}} \simeq (1 - \kappa) d_L^{\text{true}} .$$

$$\mu \approx 1 + 2\kappa ,$$

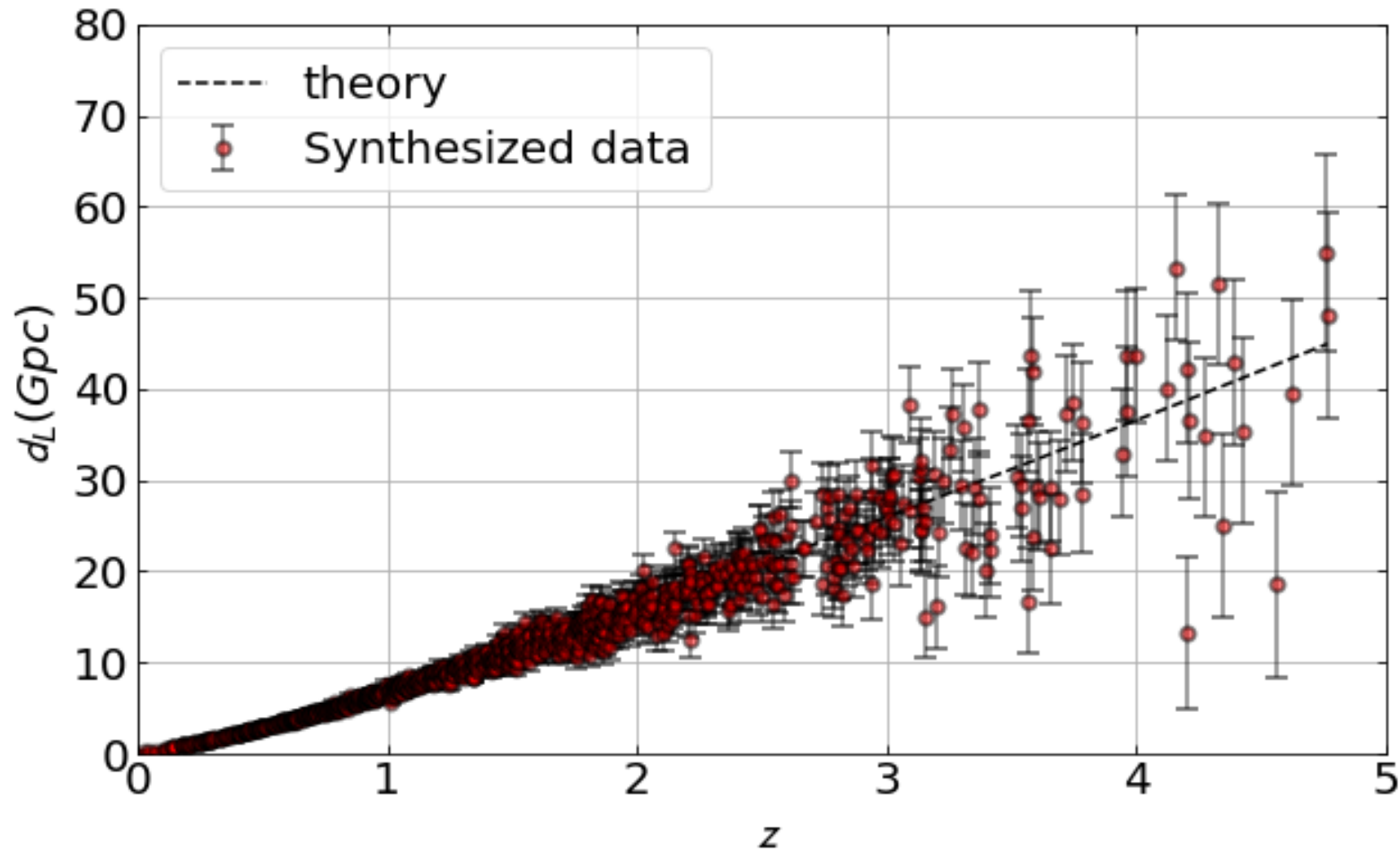
magnification of both  
electromagnetic and  
gravitational radiation  
in the geometric optics regime

بزرگ نمایی هندسی حاصله از تابش و  
امواج گرانشی



where  $\kappa$  is the lensing  
convergence, a  
weighted projection  
of density perturbations  
along the line-of-sight

# LUMINOSITY DISTANCES



$$\sigma_{d_L}^{\text{inst}} \simeq \frac{2d_L}{\rho}$$

$$\sigma_{d_L}^{\text{lens}} / d_L = 0.05z.$$

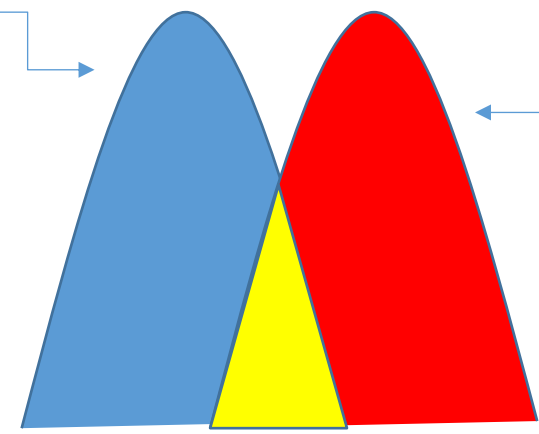
V.A

Other tools for  
constraining

GWs tool for  
constraining

Simulation with  $\Lambda$ CDM model

Observation with  $\Lambda$ CDM model

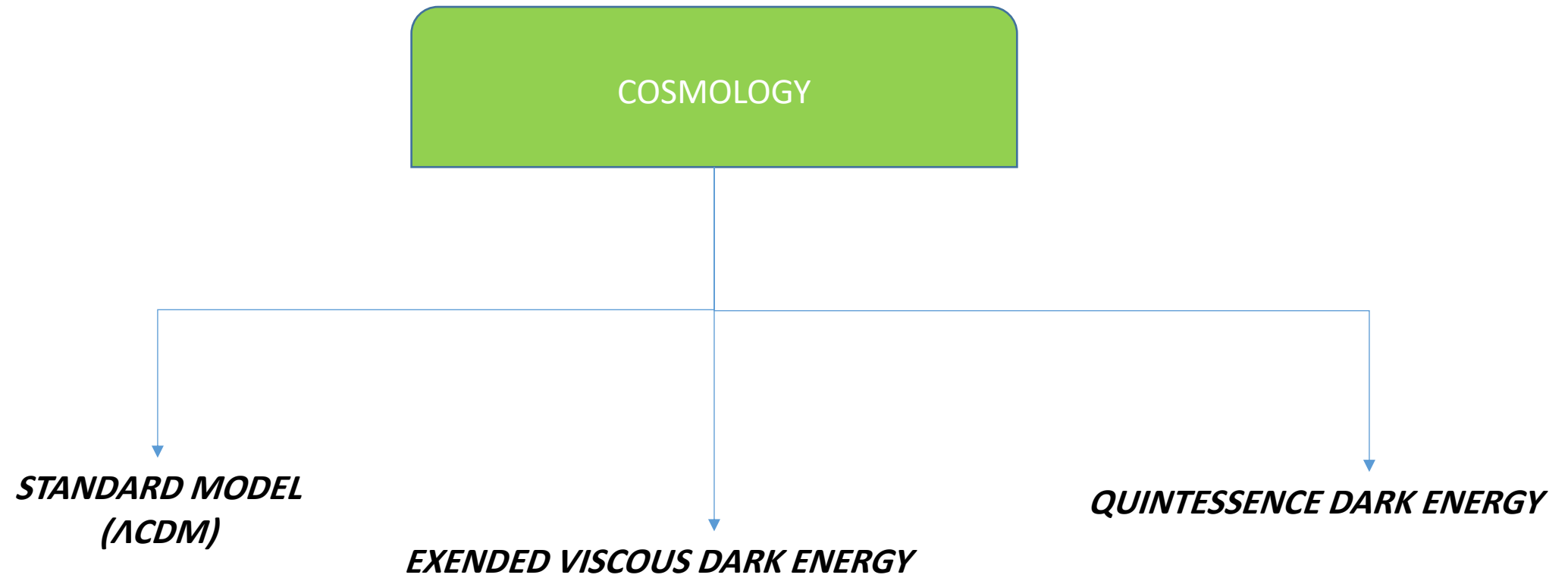


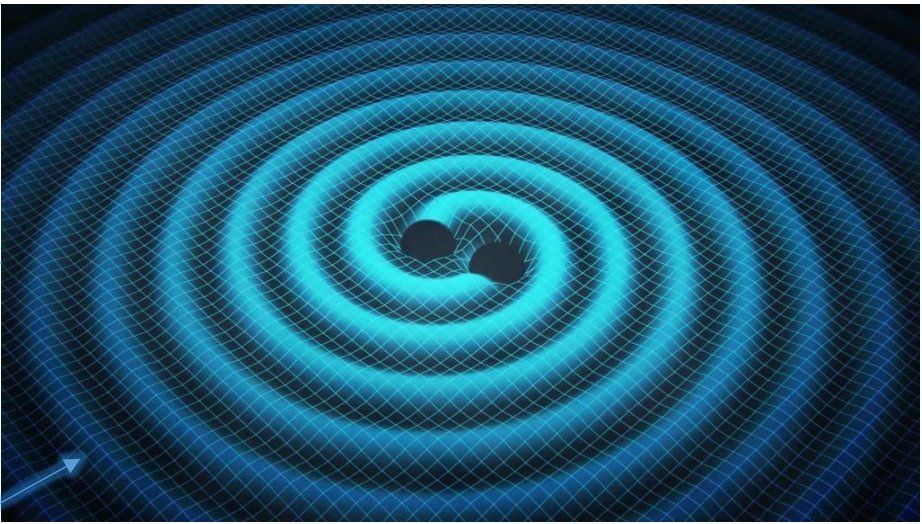
Range



DARK ENERGY

DARK ENERGY

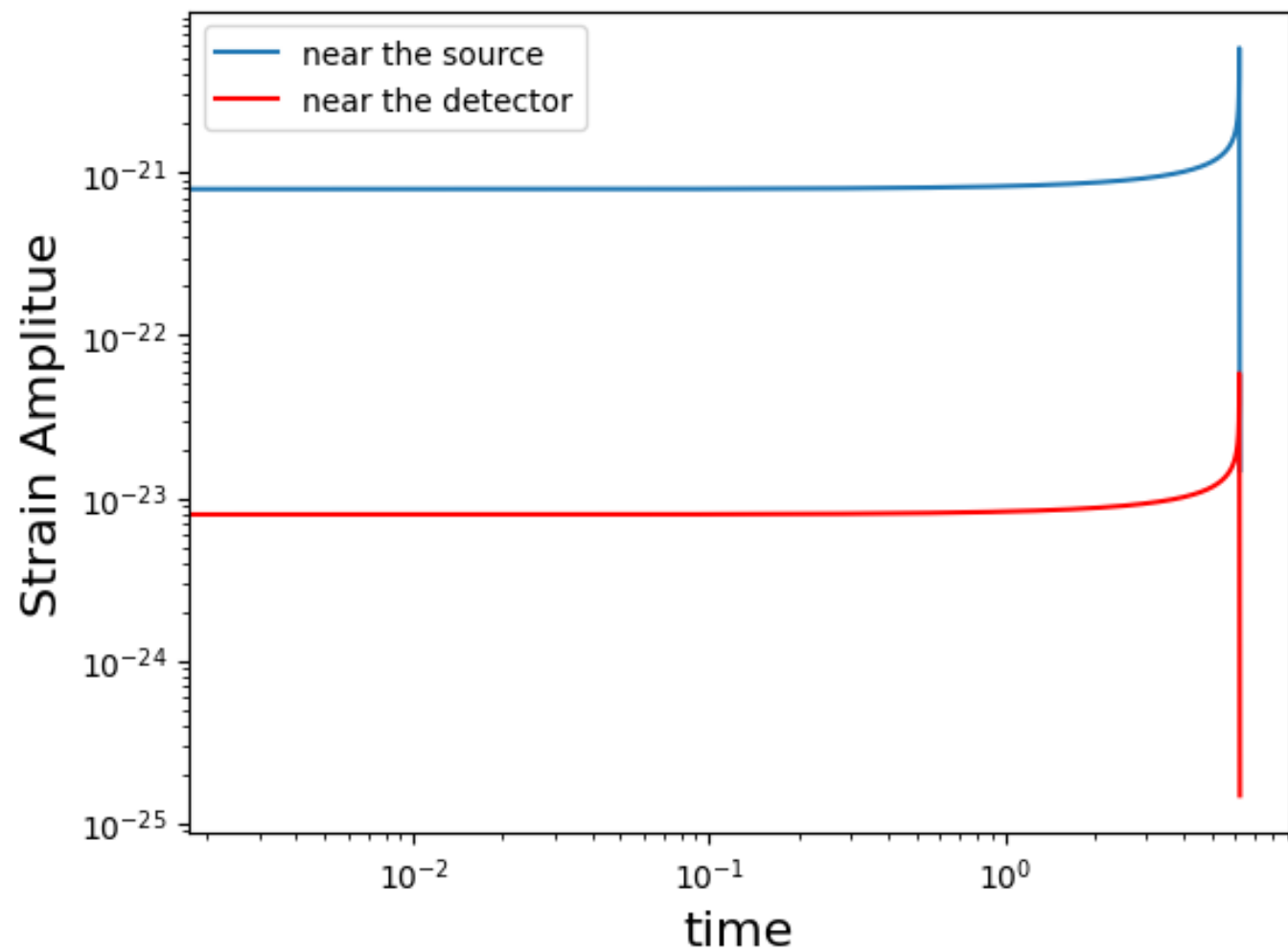




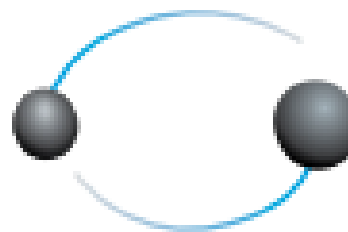
**Cosmological  
model**



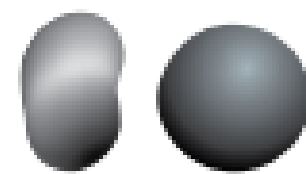
Simulation



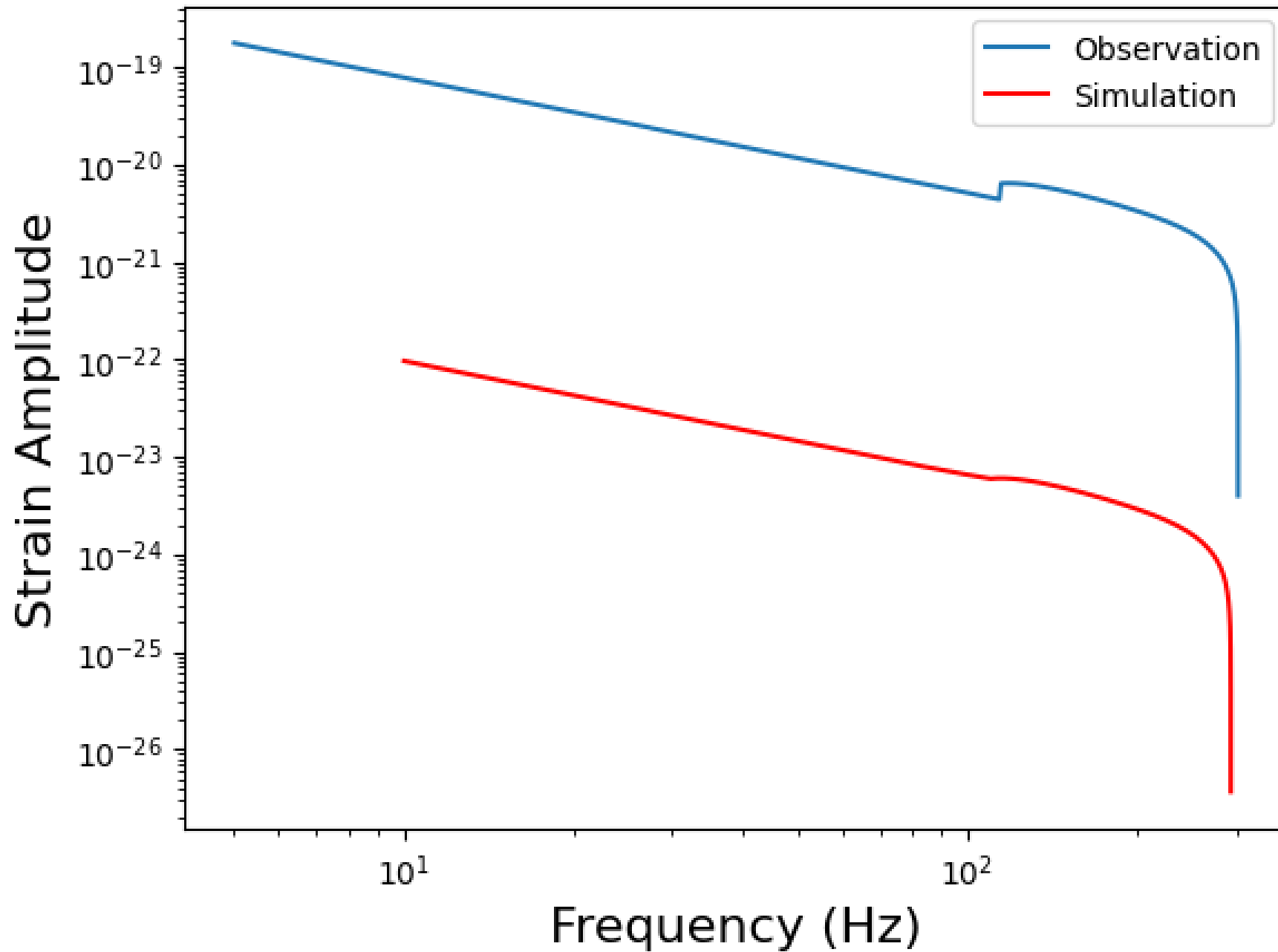
Inspiral

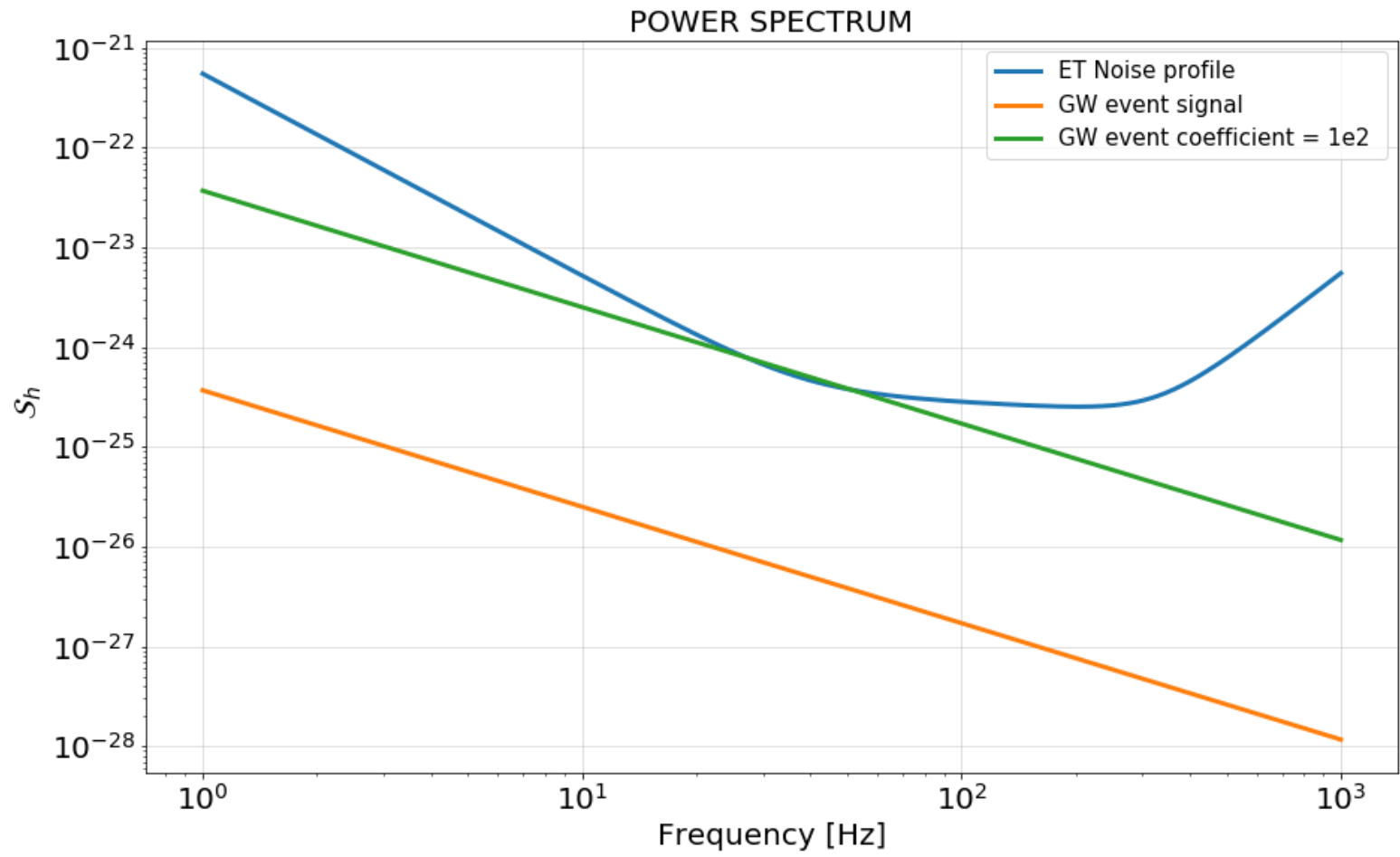


Merger Ringdown



# Binary merger





# POWER SPECTRUM

What next :

- Use different D.E models and put constrain on them.
- Complete writing section of my thesis.





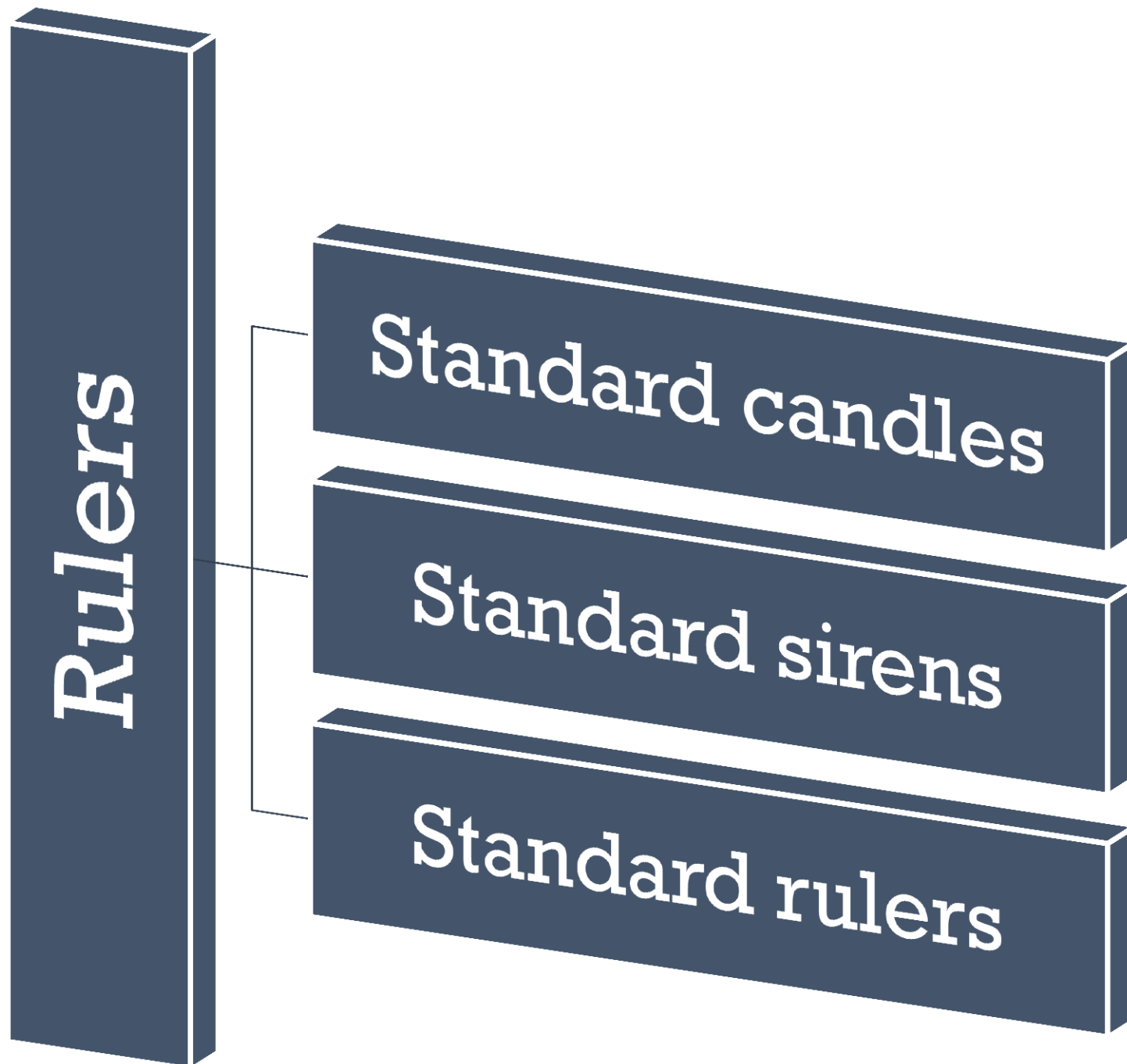
**THANKS FOR**

**YOUR ATTENTION**

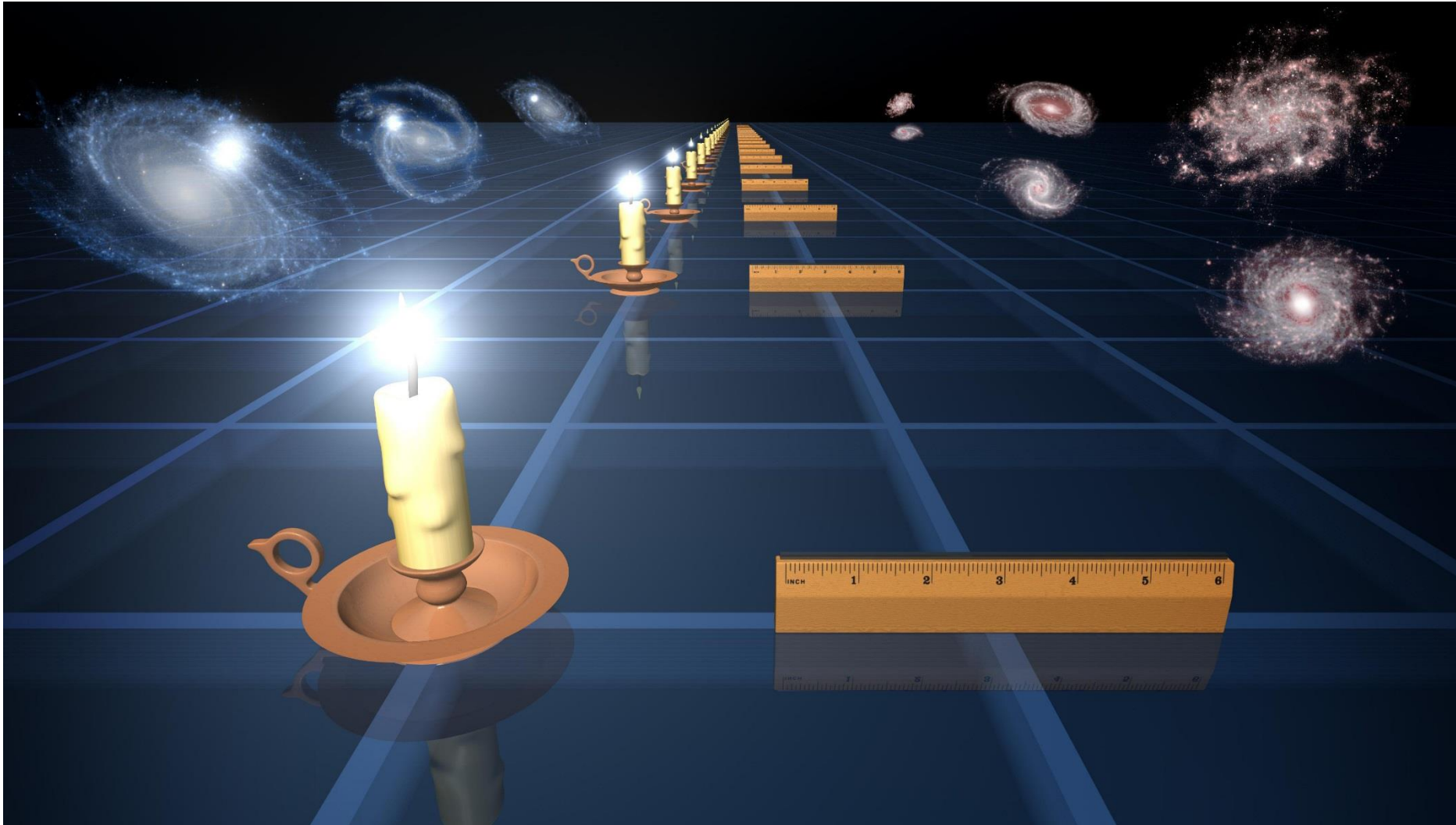


## **INTENSIVE REPORT RELATED ARTICLES**

- [1]: <https://doi.org/10.1088/1475-7516/2023/03/047>
- [2]: <https://doi.org/10.3847/2041-8213/acda9a>
- [3]: <https://doi.org/10.1038/s42254-021-00303-8>
- [4]: <https://doi.org/10.3847/2041-8213/acdac6>
- [5]: arXiv:2306.16220v2
- [6] : <http://www.njp.org/doi:10.1088/1367-2630/7/1/204>
- [7] : 10.1103/PhysRevResearch.4.013247
- [8]: EPJC-20-11-123
- [9]: <https://doi.org/10.1038/s42254-021-00303-8>
- [10] : <http://www.njp.org/doi:10.1088/1367-2630/7/1/204>
- [12]: EPJC-20-11-123
- [11] : 10.1103/PhysRevResearch.4.013247
- [13] : arXiv:1608.08008v2
- [14] : arXiv:2202.09726v3
- [15] : arXiv:2009.09754v1
- [16] : arXiv:2109.07537v2

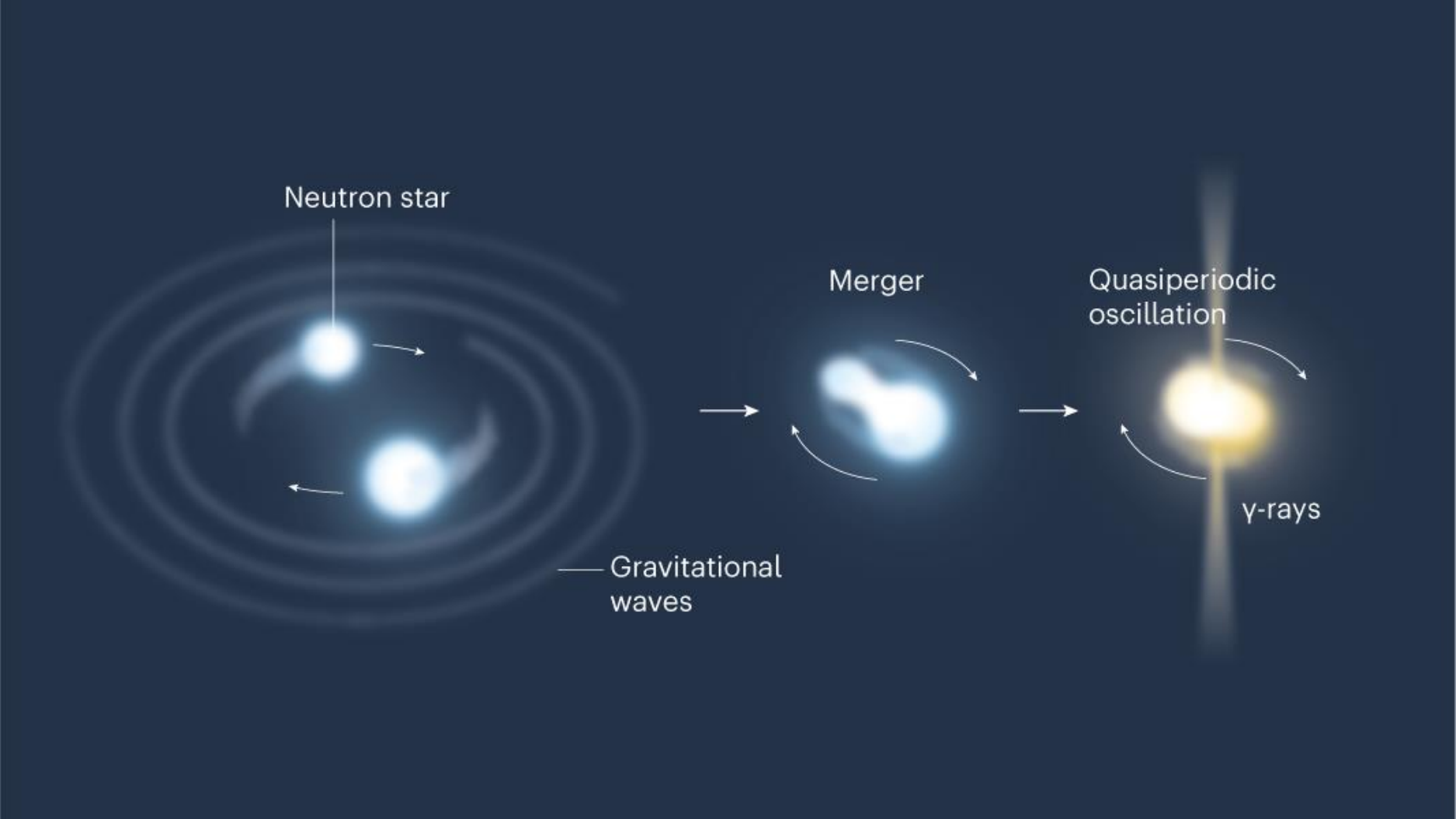


## Standard candles



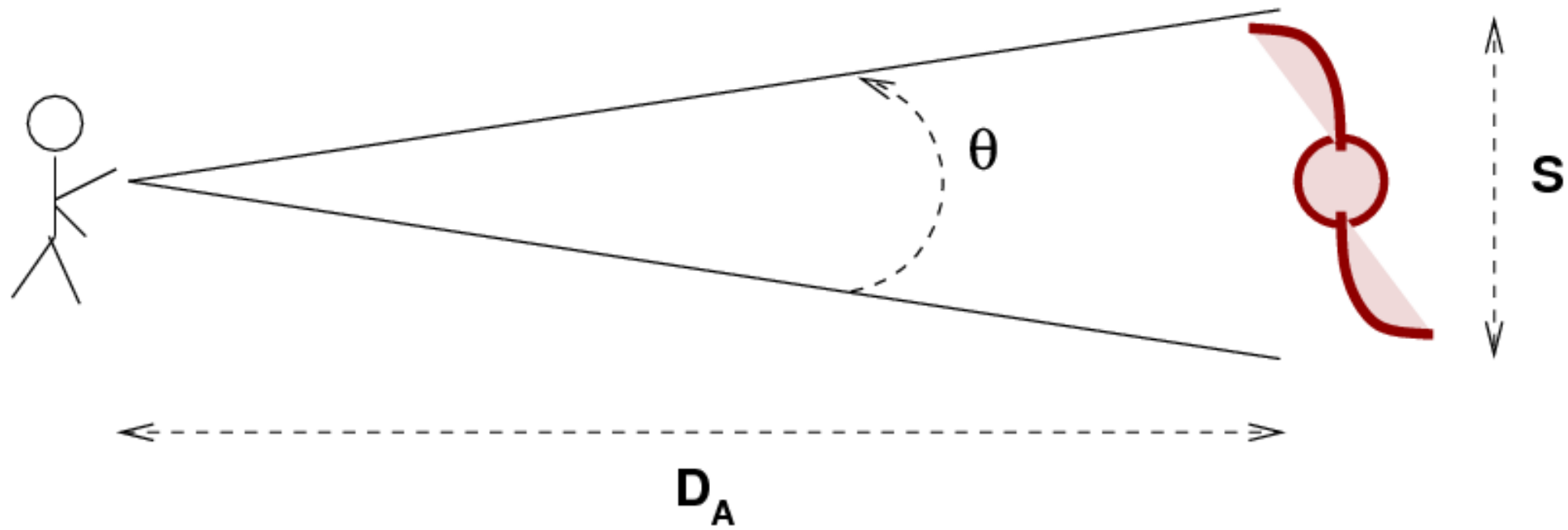
$$F = \frac{L}{4\pi d^2},$$

# Standard sirens



$$F = \frac{L}{4\pi d^2},$$

## Standard ruler



$$d_A = \frac{l}{\theta}.$$

