Quantum Chaotic Systems

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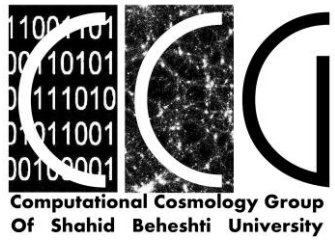

Nonlinear Dynamics and Chaos

- If x on the right-hand side appear to the first power only the equation called linear.
- Sensitive dependency on initial condition is the property of chaotic systems.
- We use Lyapunov exponent to check that is the behavior of the system is Chaotic or not.

S. H. Strogatz, Nonlinear Dynamics And Chaos: With Applications To Physics, Biology, Chemistry, And Engineering. Studies in Nonlinearity. Addison-Wesley, 1994.

Quantum and Chaos!?

- Describing chaotic dynamical systems in terms of quantum theory.
- Quantum mechanics ⟶ Linear Theory.
- Chaos ⟶ Nonlinear Theory.
- Relationship between quantum mechanics and classical chaos !?

Hilborn, Robert C. *Chaos and nonlinear dynamics: an introduction for scientists and engineers*. Oxford university press, 2000.

Can there be Quantum Chaos?

- Since we cannot identify individual trajectories in quantum mechanics, there is no any Lyapunov exponent to check chaotic behavior.

$$\Psi(t) = U(t, t_0)\,\Psi(t_0) \qquad (1)$$

$$\text{If } \Psi(t_0) + \alpha \qquad (2)$$

$$(1), (2) \longrightarrow \Psi(t) + U(t, t_0)\,\alpha$$

Hilborn, Robert C. *Chaos and nonlinear dynamics: an introduction for scientists and engineers*. Oxford university press, 2000.

Can there be Quantum Chaos?

- Two nearby wave functions remain close as time goes on.
- Exponentially diverging nearby trajectories to quantum mechanics, as a possible test for quantum chaos, will not work.
- We believe that this argument about the lack of exponential divergence in quantum mechanics is correct.

Hilborn, Robert C. *Chaos and nonlinear dynamics: an introduction for scientists and engineers*. Oxford university press, 2000.

Can there be Quantum Chaos?

- Quantum model ⟶ Backward integration is possible.
- Classical chaotic systems ⟶ Backward integration is not possible because of the existence of a positive Lyapunov exponent.
- We cannot reverse the time integration to recover the initial distribution for a truly chaotic classical systems.

Hilborn, Robert C. *Chaos and nonlinear dynamics: an introduction for scientists and engineers*. Oxford university press, 2000.

Experiments on Quantum Chaos

- Experiments shows that the systems whose classical analogs show chaotic behavior have results that are in agreement with the predictions of quantum mechanics.
- An atom with a single electron which is placed in a strong magnetic filed.
- Interaction of lasers and atoms.
- And….

Hilborn, Robert C. *Chaos and nonlinear dynamics: an introduction for scientists and engineers*. Oxford university press, 2000.

Types of Quantum Chaos

- Quantized chaos ⟶ Known as quantum chaology.
- Also is the most frequently studied form of quantum chaos.
- All atoms and molecules except Hydrogen atom and related two-body systems exhibit chaos when treated classically.
- Semi-quantum chaos.
- Genuine quantum chaos.

Porter, Mason A. "An introduction to quantum chaos." *arXiv preprint nlin/0107039* (2001).

What is the Open question?

- Finding a signature for quantum chaos in different systems (Quantum Kicked-Top (QKT), Quantum Kicked-Rotor (QKR) and…).
- **Entanglement Entropy as a signature of quantum chaos.**
- Non-Gaussianity as a signature of quantum chaos.
- And…

Piga, Angelo, Maciej Lewenstein, and James Q. Quach. "Quantum chaos and entanglement in ergodic and nonergodic systems." *Physical Review E* 99.3 (2019): 032213.

Entanglement Entropy as a signature of quantum chaos

- Entanglement entropy (EE) as a signature of quantum chaos in ergodic and nonergodic systems.
- More recently bipartite EE as a signature of chaos was studied in the quantum kicked top (QKT) modeled as a multispin system.
- Can EE be a signature of quantum chaos in highly nonergodic systems?

Piga, Angelo, Maciej Lewenstein, and James Q. Quach. "Quantum chaos and entanglement in ergodic and nonergodic systems." *Physical Review E* 99.3 (2019): 032213.

Quantum Kicked Top System

- The Hamiltonian of the QKT is:

$$H_T = \alpha J_x + \frac{\beta}{j} J_z^2 \sum_{n=-\infty}^{\infty} \delta(t - n) \quad (1)$$

- The first term in Eq. (1) describes a precession around the x axis with angular frequency α. The second term represents a periodic kick.
- Does this notion of chaos and ergodicity hold in the quantum case?

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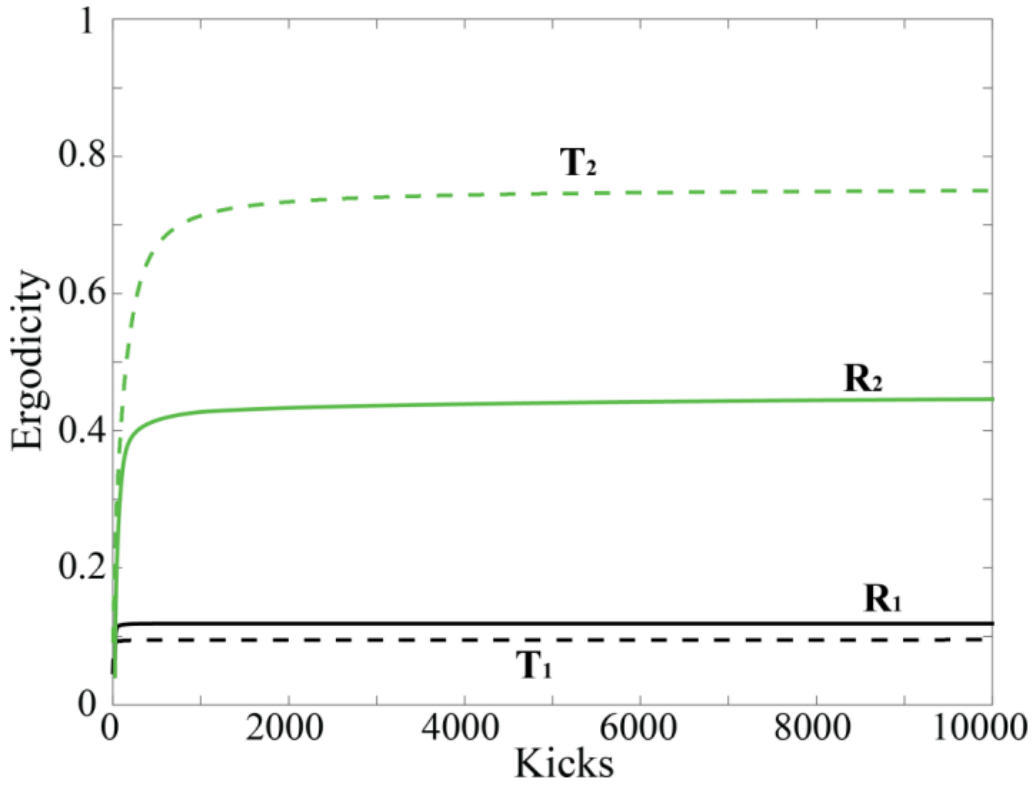
Ergodicity-Kicks



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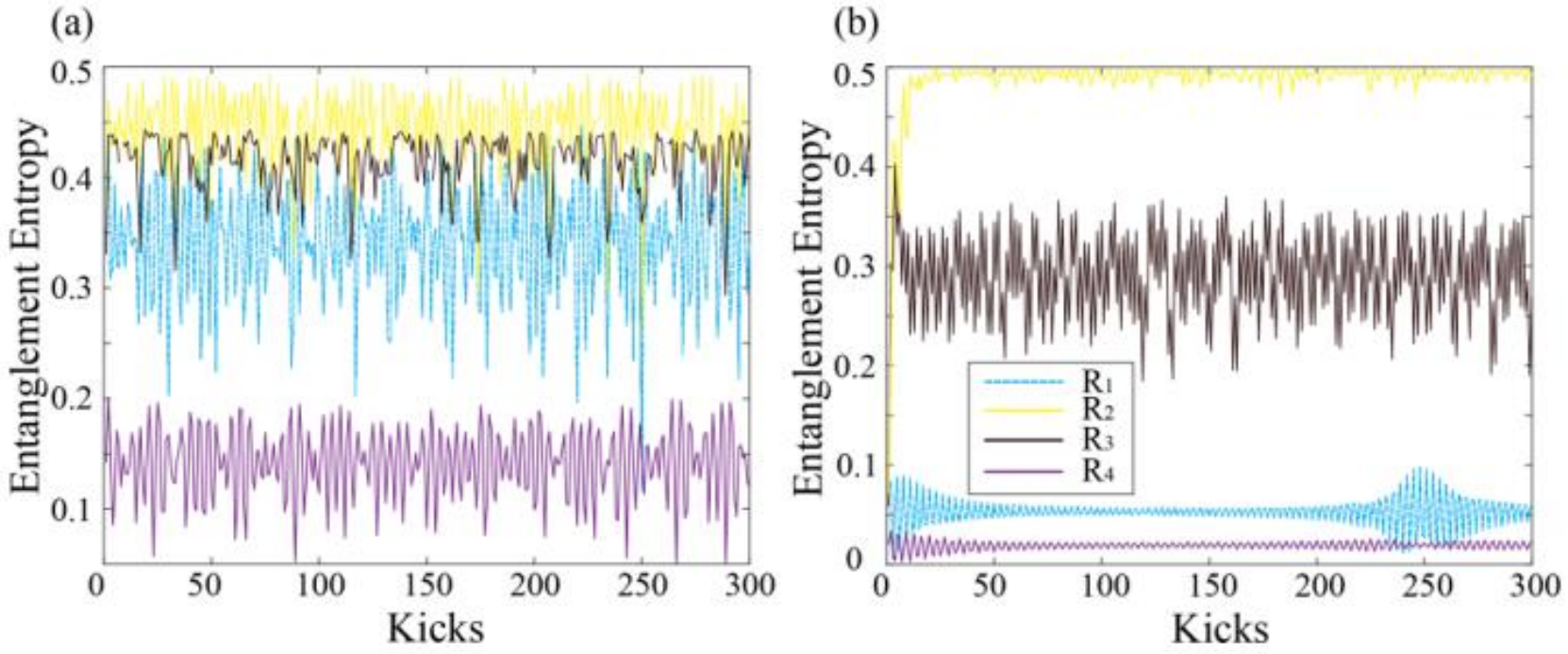
Entanglement-Entropy



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Conclusion

- We have explored the correspondence of EE and chaos in the kicked top.
- High EE corresponds to global chaos in ergodic systems.
- EE corresponds to local chaos in nonergodic systems.

Piga, Angelo, Maciej Lewenstein, and James Q. Quach. "Quantum chaos and entanglement in ergodic and nonergodic systems." *Physical Review E* 99.3 (2019): 032213.

Another interesting avenue of future investigation

- Understand what the role of bipartition choice plays in quantum chaos.
- At the interface of experimental accessibility and theoretical analysis, the QKT and QKR continue to be fruitful systems in which to study quantum chaos.

Piga, Angelo, Maciej Lewenstein, and James Q. Quach. "Quantum chaos and entanglement in ergodic and nonergodic systems." *Physical Review E* 99.3 (2019): 032213.

Thanks for your attention

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