Intensive
Scientific
Report

Investigating Stochastic Gravitational Wave Background as a Cosmological and Astrophysical Probe

PhD research in gravitation and cosmology

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Supervisor: Dr. Seyed Mohammad Sadegh Movahed

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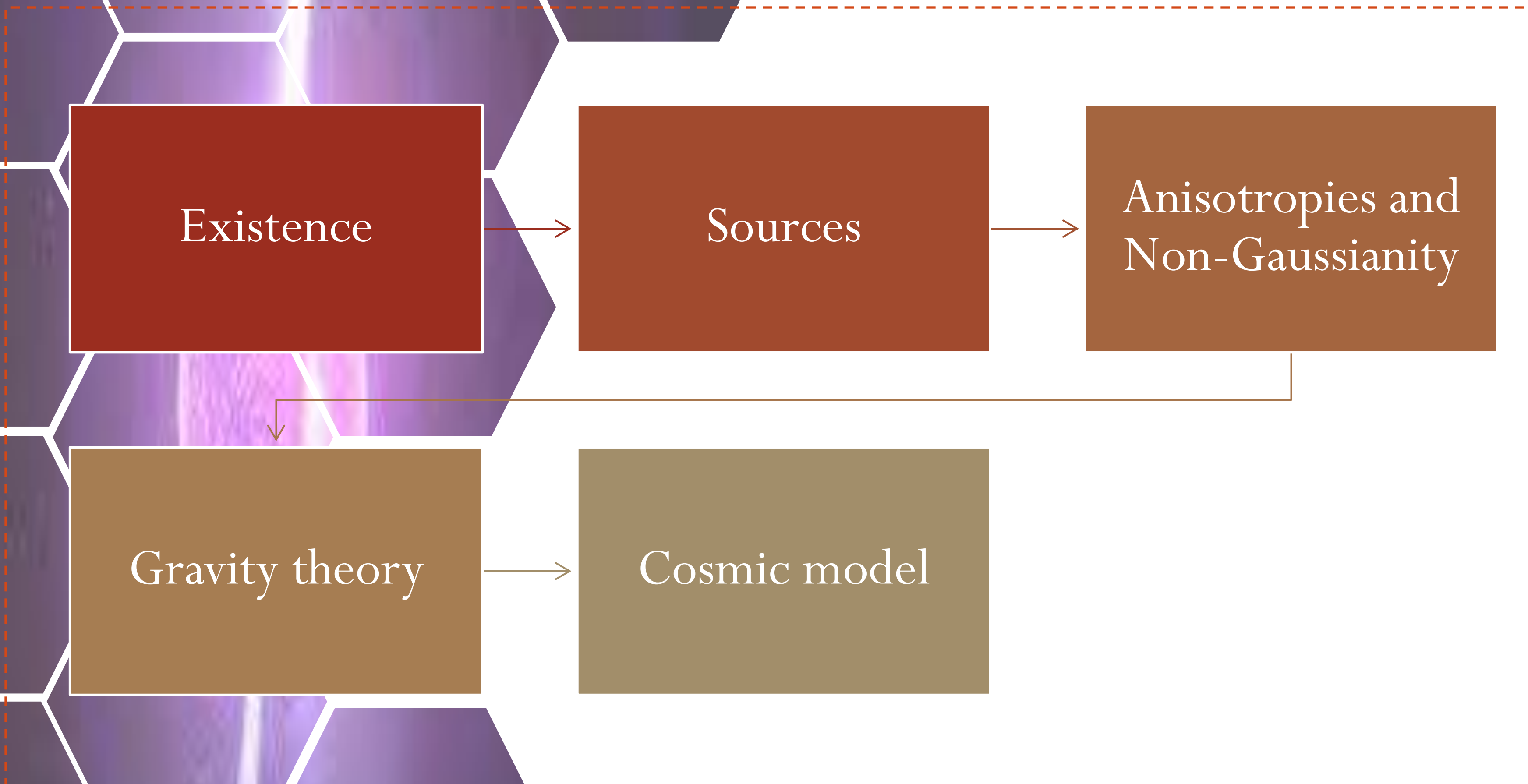
Road map


Existence
Sources
Anisotropies and Non-Gaussianity
Gravity theory
Cosmic model

Road map

Existence

Sources

Anisotropies and Non-Gaussianity

Cosmic model

- ✓ Different SNRs
- ✓ Different signal components
- ✓ Different ORF

Road map

Existence → Sources → Anisotropies and Non-Gaussianity

- ✓ Different SNRs
- ✓ Different signal compone
- ✓ Different ORF

- ✓ Cosmic sources
- ✓ Astrophysical sources

Cosmic model

Road map

Existence → Sources → Anisotropies and Non-Gaussianity

Gravity theory → Cosmic model

- ✓ Cosmic sources
- ✓ Astrophysical sc

- ✓ Angular anisotropies
- ✓ Primordial non-gaussianity

Road map

Existence → Sources → Anisotropies and Non-Gaussianity

- ✓ Angular anisotropies
- ✓ Primordial non-gaussianity

Gravity theory → Cosmic model

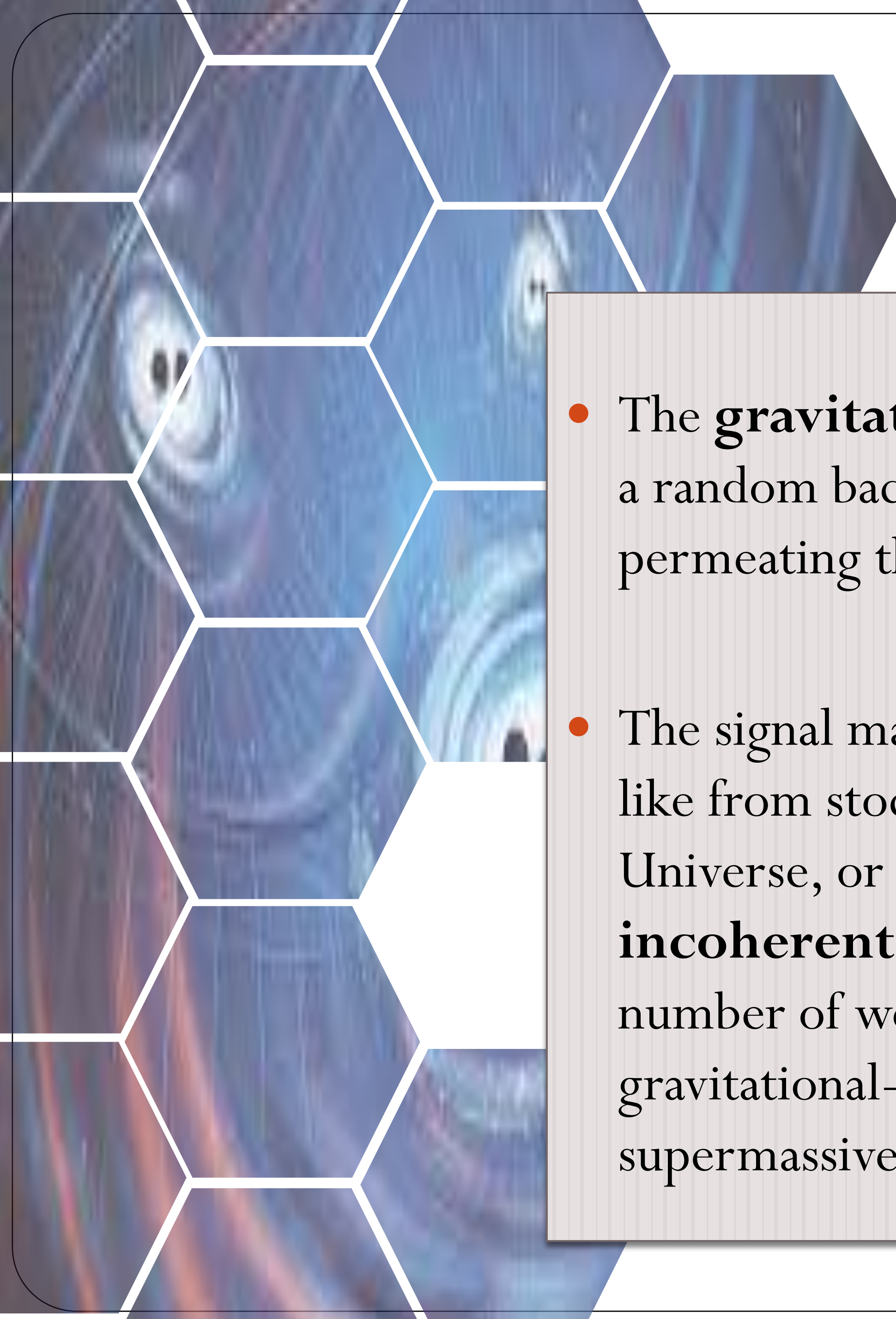
- ✓ General relativity
- ✓ Alternative theories

Road map

Existence → Sources → Anisotropies and Non-Gaussianity → Gravity theory → Cosmic model

- ✓ General relativity
- ✓ Alternative theories

- ✓ ΛCDM
- ✓ Alternative model



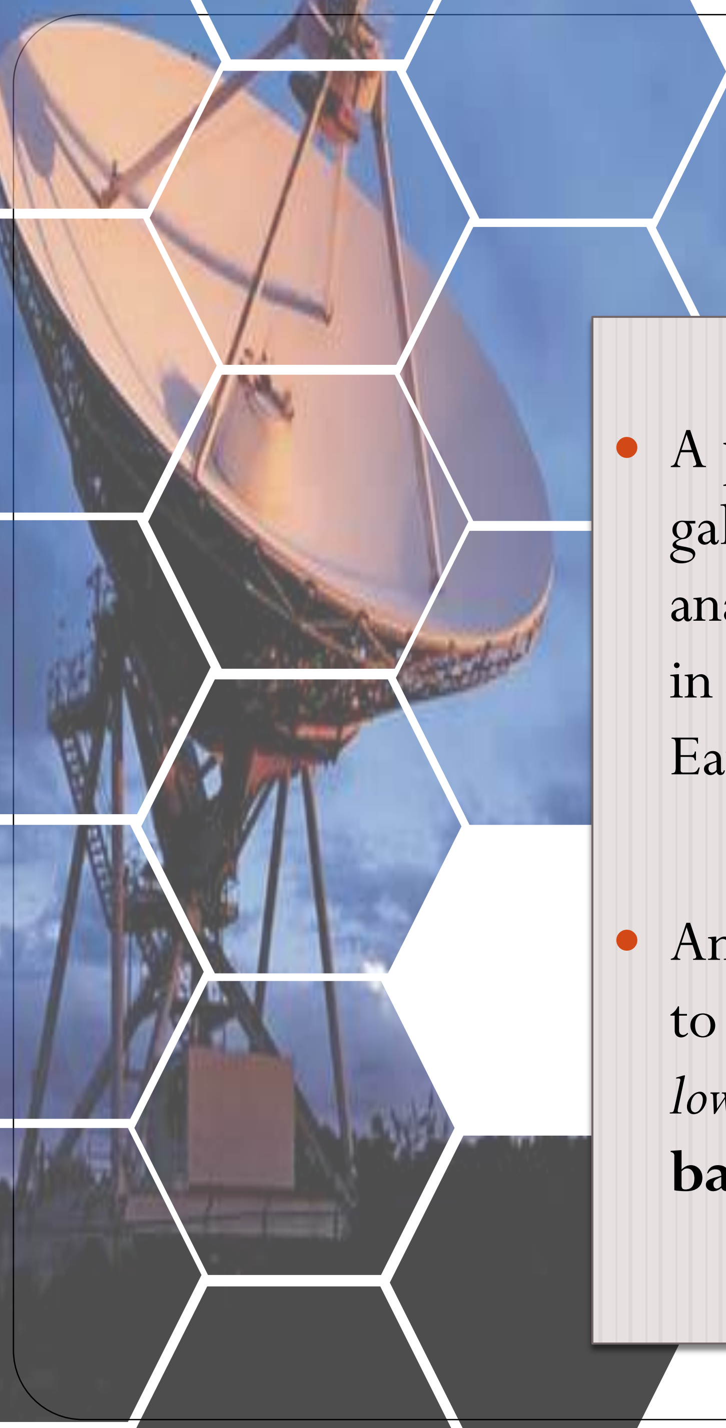
GWB

- The **gravitational wave background** is a random background of gravitational waves permeating the Universe.

- The signal may be **intrinsically random**, like from stochastic processes in the early Universe, or may be produced by an **incoherent superposition** of a large number of weak independent unresolved gravitational-wave sources, like supermassive black-hole binaries.

GWB

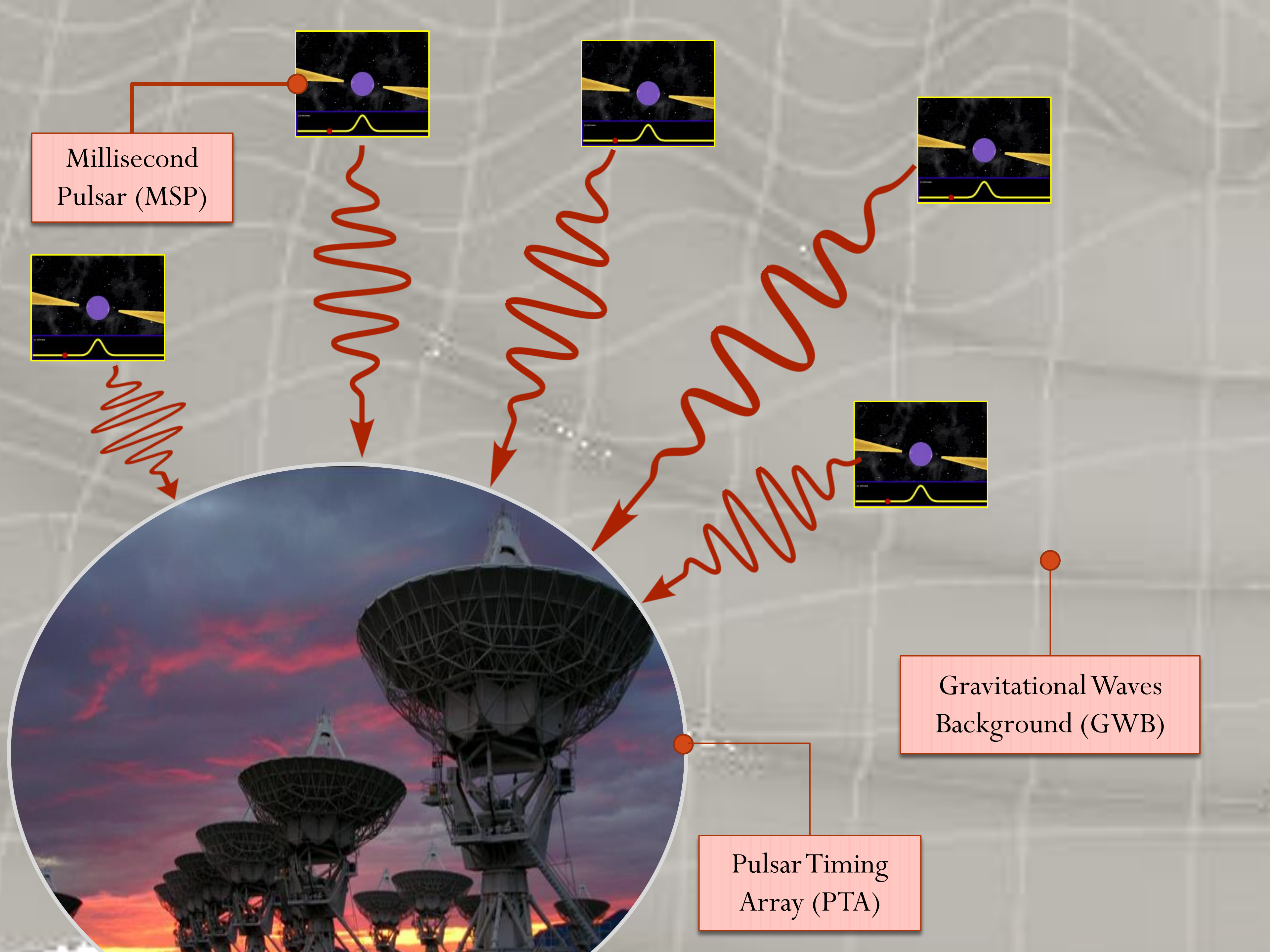
- **Cosmological sources**
  - inflationary scalar fields
  - cosmological phase transitions
  - cosmic strings
- **Astrophysical sources**
  - binary black-hole mergers
  - the final moments of an explosive supernova event
  - rapidly rotating neutron stars



PTAs

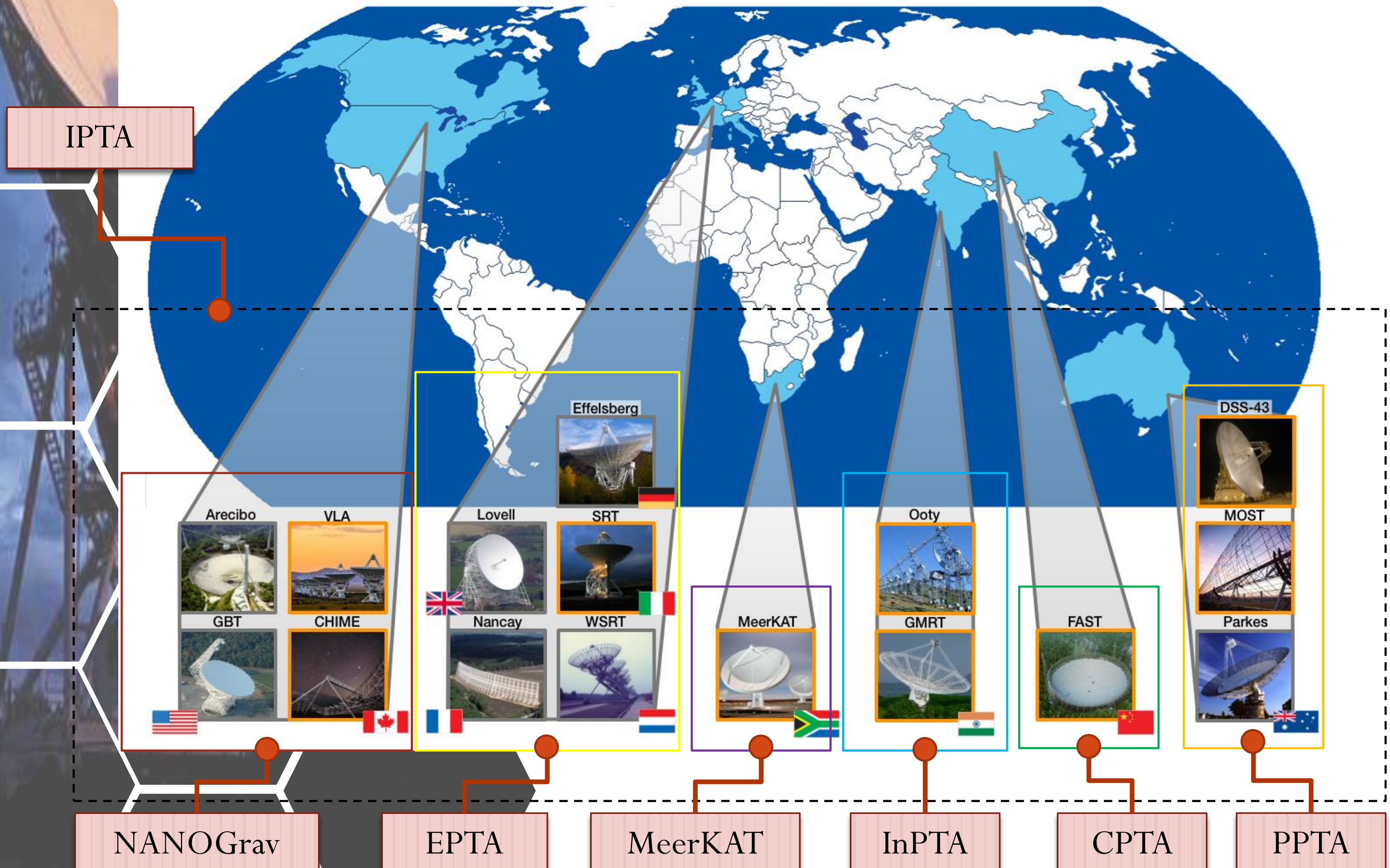
- A **pulsar timing array** (**PTA**) is a set of galactic pulsars that is monitored and analyzed to search for correlated 'signatures in the **pulse arrival times (TOAs)** on Earth.

- An array of **millisecond pulsars (MSPs)** to detect and analyze *long-wavelength (i.e., low-frequency)* **gravitational waves background (GWB)**.

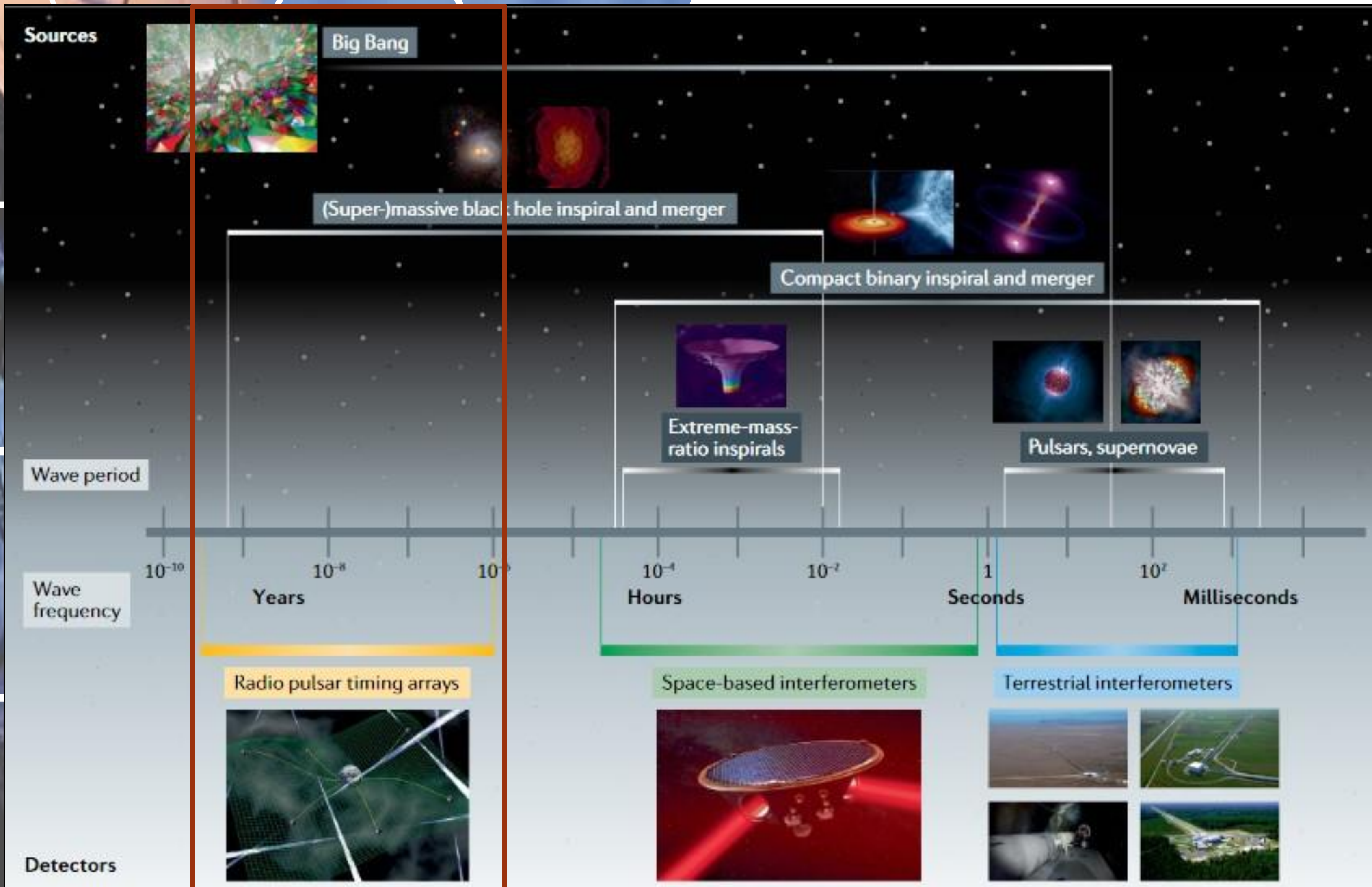


Millisecond Pulsar (MSP)
Gravitational Waves Background (GWB)
Pulsar Timing Array (PTA)

PTAs


IPTA
Arecibo
VLA
GBT
CHIME
Effelsberg
Lovell
SRT
Nancay
WSRT
MeerKAT
Ooty
GMRT
FAST
DSS-43
MOST
Parkes
NANOGrav
EPTA
MeerKAT
InPTA
CPTA
PPTA

PTAs



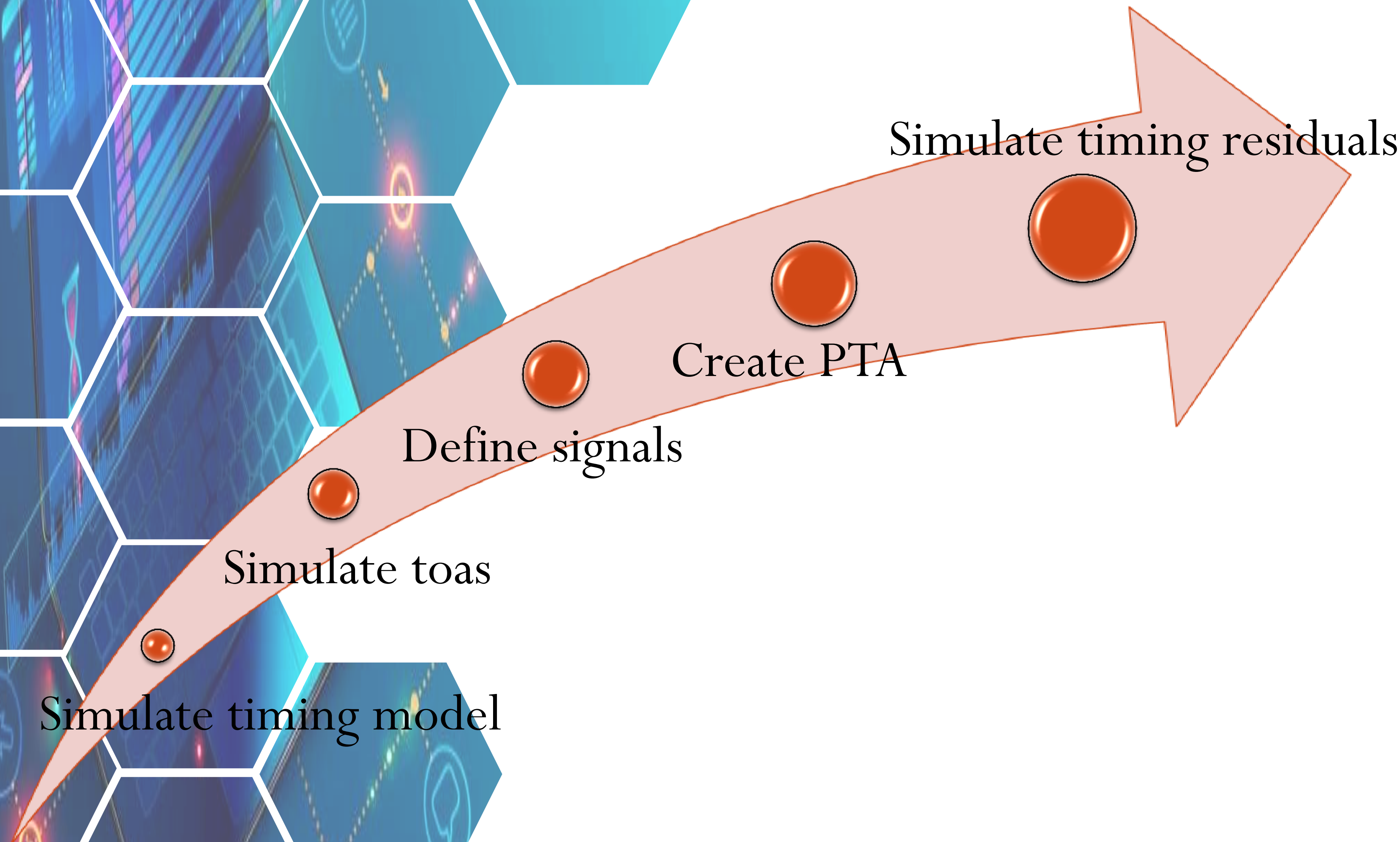
Sources
Big Bang
(Super-)massive black hole inspiral and merger
Compact binary inspiral and merger
Extreme-mass-ratio inspirals
Pulsars, supernovae
Wave period
Wave frequency
10^{-10}
10^{-8}
10
10^{-4}
10^{-2}
1
10^{2}
Years
Hours
Seconds
Milliseconds
Radio pulsar timing arrays
Space-based interferometers
Terrestrial interferometers
Detectors

PTA signals

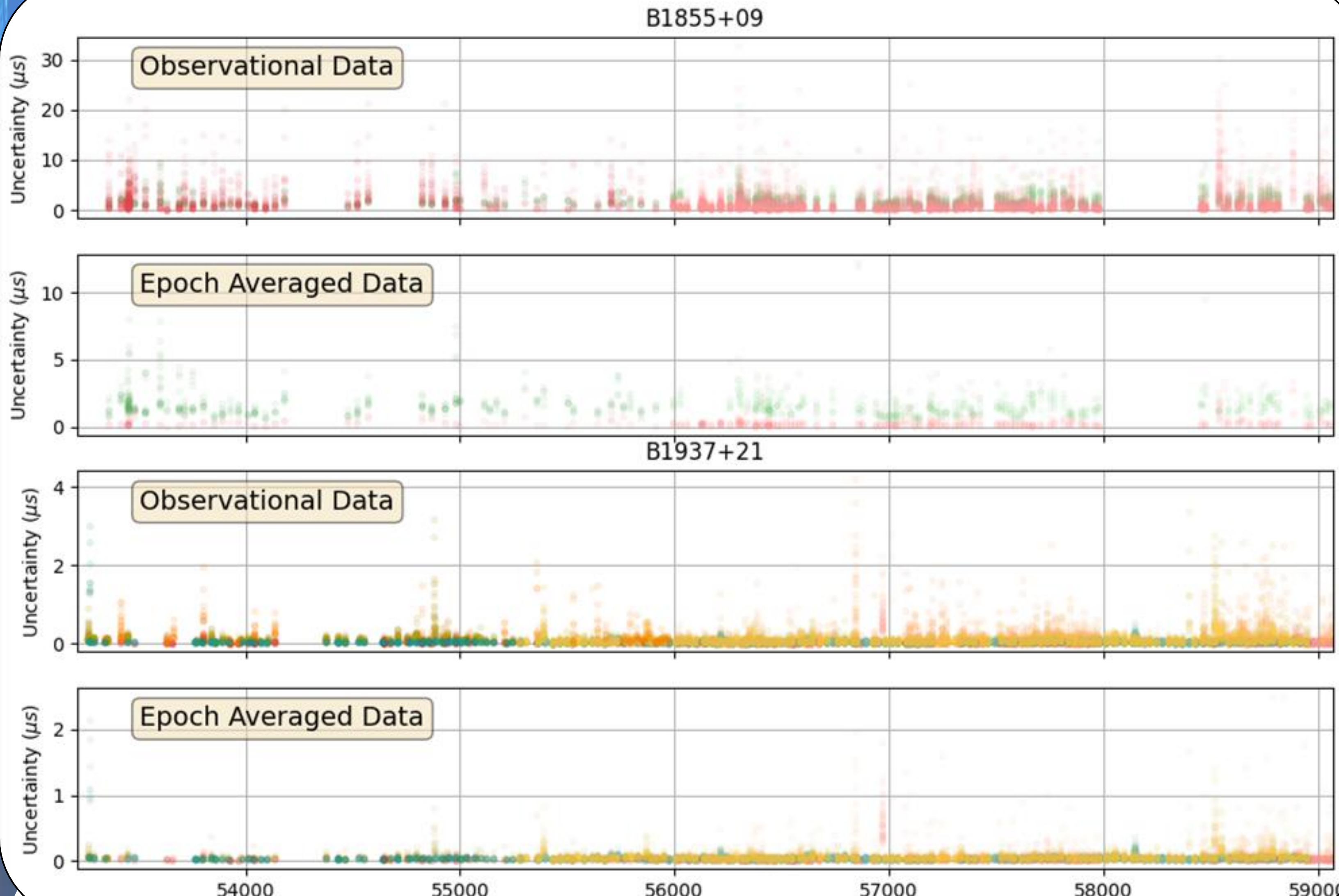
$$R_x^k(t) = \sum_{\alpha=1}^{M} M_{x,\alpha}(t) \cdot \epsilon_{x,\alpha}^k + \sum_{A=1}^{2N_f} F_{x,A}(t) \cdot a_{x,A}^k + n_x^k(t)$$

$$F_{x,A}(t) = \begin{cases} \cos(2\pi f_A t) & if\ A\ is\ odd \\ \sin(2\pi f_A t) & if\ A\ is\ even \end{cases}$$

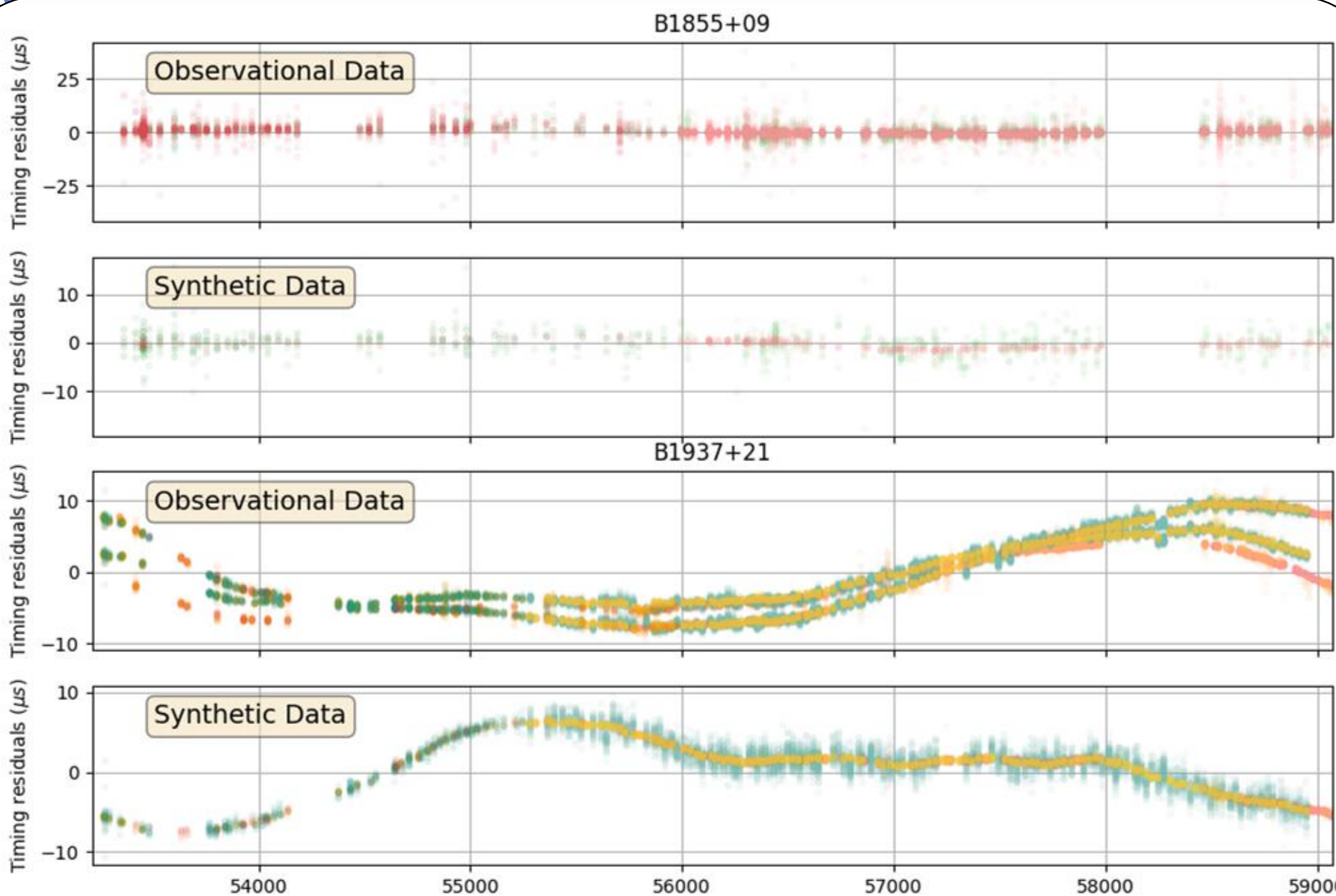
Data Simulation



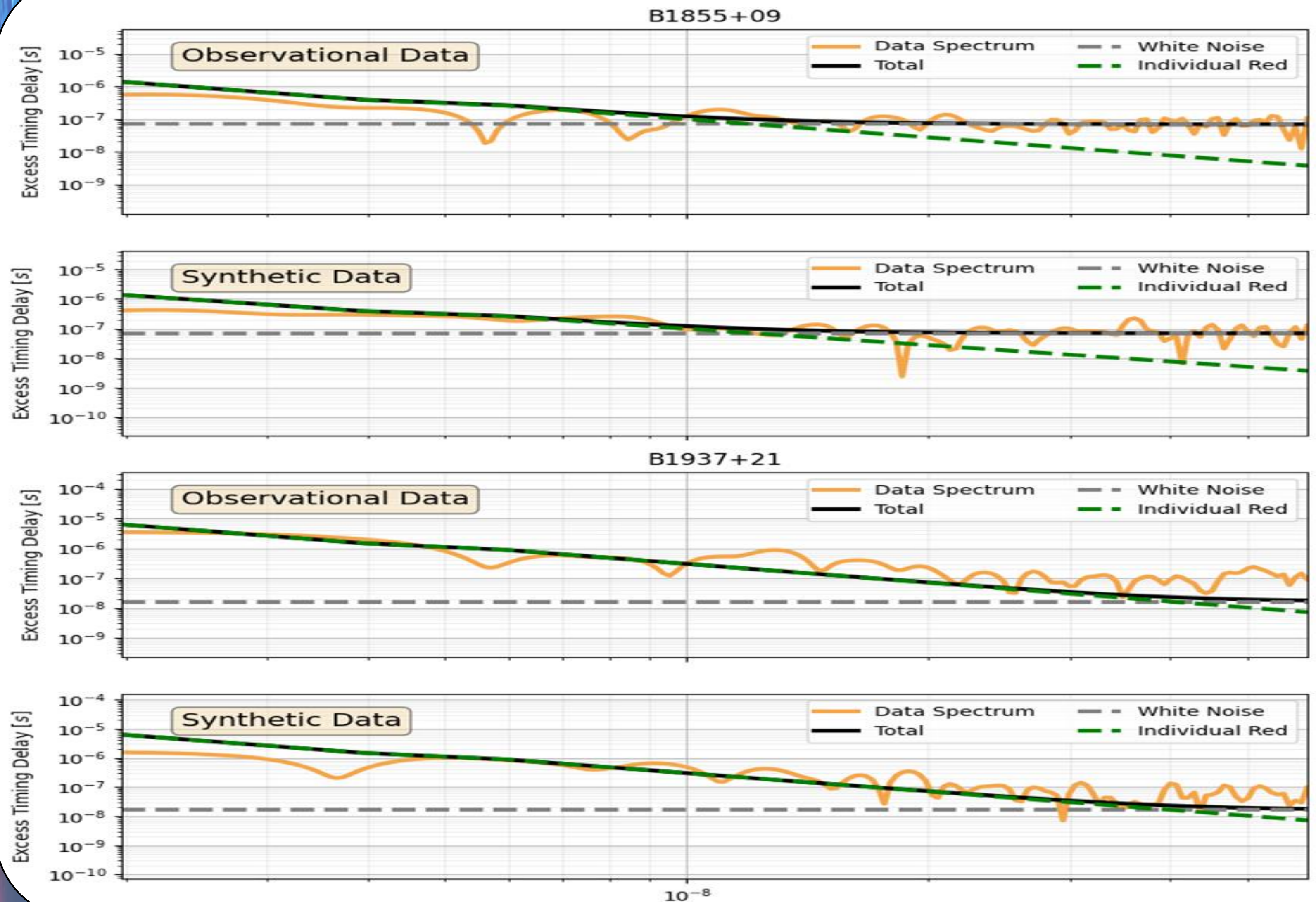
Data Simulation



Data Simulation



Data Simulation



Data Analysis

Unweighted-TPCF

$$\psi_{cr-cr} = \frac{\langle n^k_{cr,x} n^k_{cr,y} \rangle}{\langle n^k_{cr,x} \rangle \langle n^k_{cr,y} \rangle} - 1$$

$$\begin{aligned} n^k_{cr,x}(f_x) &= \delta(\tilde{R}^k_x(t) - f_x) \left| \partial_t \tilde{R}^k_x \right| \\ n^k_{cr,y}(f_y) &= \delta(\tilde{R}^k_y(t) - f_y) \left| \partial_t \tilde{R}^k_y \right| \end{aligned}$$

Data Analysis

Unweighted-TPCF

$$\tilde{R}_x^k(t) = R_x^k(t) - \sum_{\alpha=1}^{M} M_{x,\alpha} \cdot \epsilon_x^k$$

$$\mathcal{A} = \left(\tilde{R}_x^k(t), \tilde{R}_y^k(t), \partial_t \tilde{R}_x^k(t), \partial_t \tilde{R}_y^k(t)\right)$$

$$P(\mathcal{A}) = \frac{1}{\sqrt{(2\pi)^4 \det(\mathcal{K}_2)}} \exp\left(-\frac{\mathcal{A} \cdot \mathcal{K}_2^{-1} \cdot \mathcal{A}^T}{2}\right)$$

Data Analysis

Unweighted-TPCF

$$\mathcal{K}_2 = \begin{pmatrix} \sigma_{0x}^2 & C_{xy} & 0 & 0 \\ C_{xy} & \sigma_{0y}^2 & 0 & 0 \\ 0 & 0 & \sigma_{1x}^2 & D_{xy} \\ 0 & 0 & D_{xy} & \sigma_{1y}^2 \end{pmatrix}$$

$$\sigma_{0x}^2 = \left\langle \tilde{R}_x^k(t)\tilde{R}_x^k(t) \right\rangle_{ens}$$

$$\sigma_{1x}^2 = \left\langle \partial_t\tilde{R}_x^k(t)\partial_t\tilde{R}_x^k(t) \right\rangle_{ens}$$

$$C_{xy} = \left\langle \tilde{R}_x^k(t)\tilde{R}_y^k(t) \right\rangle_{ens}$$

$$D_{xy} = \left\langle \partial_t\tilde{R}_x^k(t)\partial_t\tilde{R}_y^k(t) \right\rangle_{ens}$$

Data Analysis

Unweighted-TPCF

$$\langle n_{cr,x} n_{cr,y} \rangle (f_x, f_y) = \int n^k_{cr,x}(f_x) n^k_{cr,y}(f_y) P(\mathcal{A}) d\mathcal{A}$$

$$\langle n_{cr,x} n_{cr,y} \rangle = \int \langle n_{cr,x} n_{cr,y} \rangle (f_x, f_y) df_x df_y$$

Data Analysis

Unweighted-TPCF

$$
\begin{aligned}
\sigma_{0x}^2 &= \sum_{A=1}^{2N_f} F_{x,A}(t) \cdot \langle a_{x,A}^k a_{x,A}^k \rangle_{ens} \cdot F_{x,A}(t) + \langle n_x^k(t) n_x^k(t) \rangle_{ens} \\
&= \sum_{A=1}^{2N_f} F_{x,A}(t) \cdot \phi_{x,rn}(f_A) \cdot F_{x,A}(t) \\
&\quad + \sum_{A=1}^{2N_f} F_{x,A}(t) \cdot \phi_{gw}(f_A) \cdot F_{x,A}(t) + \langle n_x^k(t) n_x^k(t) \rangle_{ens}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{1x}^2 &= \sum_{A=1}^{2N_f} \partial_t F_{x,A}(t) \cdot \langle a_{x,A}^k a_{x,A}^k \rangle_{ens} \cdot \partial_t F_{x,A}(t) + \langle \partial_t n_x^k(t) \partial_t n_x^k(t) \rangle_{ens} \\
&= \sum_{A=1}^{2N_f} \partial_t F_{x,A}(t) \cdot \phi_{x,rn}(f_A) \cdot \partial_t F_{x,A}(t) \\
&\quad + \sum_{A=1}^{2N_f} \partial_t F_{x,A}(t) \cdot \phi_{gw}(f_A) \cdot \partial_t F_{x,A}(t) + \langle \partial_t n_x^k(t) \partial_t n_x^k(t) \rangle_{ens}
\end{aligned}
$$

Data Analysis

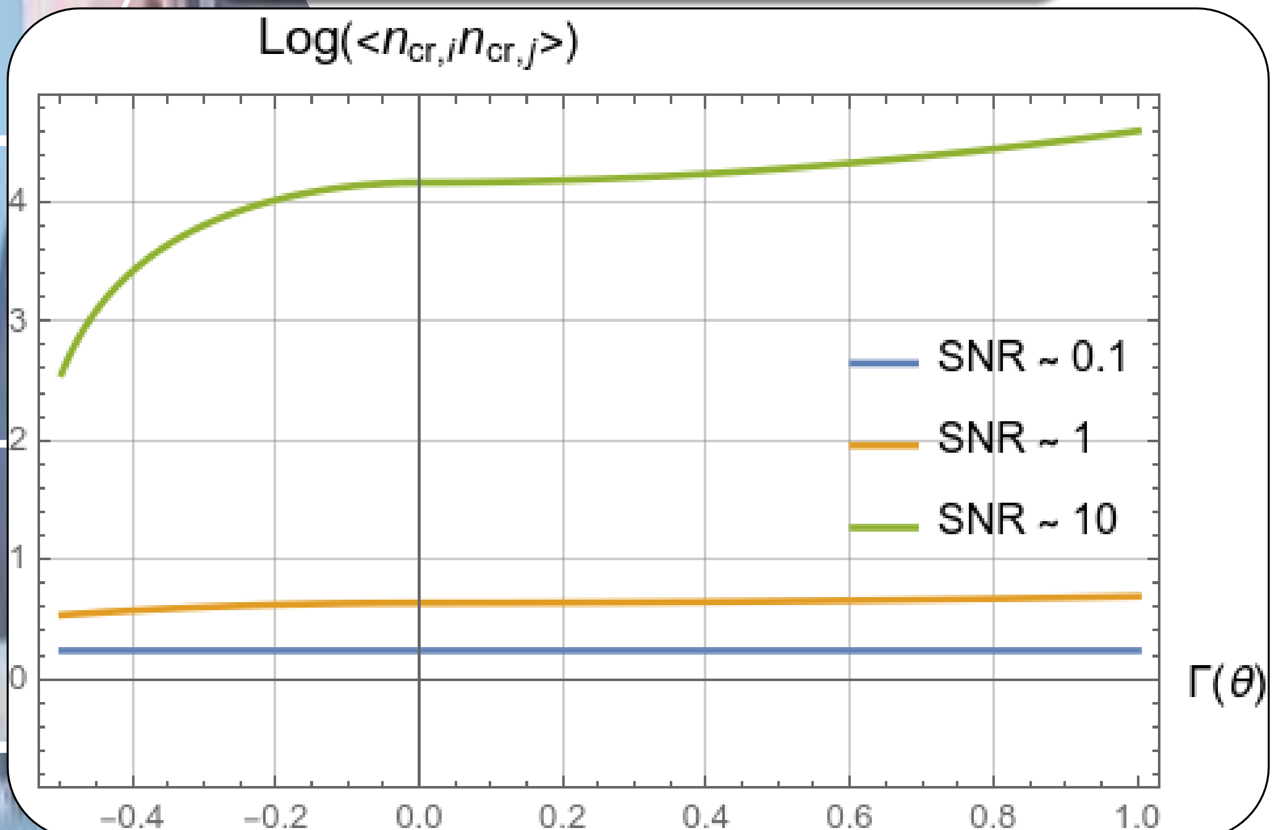
Unweighted-TPCF

$$
\begin{aligned}
C_{xy} &= \sum_{A=1}^{2N_f} F_{x,A}(t) \cdot \langle a^k_{x,A} a^k_{y,A} \rangle_{ens} \cdot F_{y,A}(t) \\
&= \sum_{A=1}^{2N_f} F_{x,A}(t) \cdot \phi_{gw}(f_A) \Gamma(\theta_{xy}) \cdot F_{y,A}(t)
\end{aligned}
$$

$$
\begin{aligned}
D_{xy} &= \sum_{A=1}^{2N_f} \partial_t F_{x,A}(t) \cdot \langle a^k_{x,A} a^k_{y,A} \rangle_{ens} \cdot \partial_t F_{y,A}(t) \\
&= \sum_{A=1}^{2N_f} \partial_t F_{x,A}(t) \cdot \phi_{gw}(f_A) \Gamma(\theta_{xy}) \cdot \partial_t F_{y,A}(t)
\end{aligned}
$$

Data Analysis

Unweighted-TPCF



What's next?

- Calculate another geometrical measures
- TDA
- Add gw signals
- Change the ORF
- Search for anisotropies and/or non-gaussianity
- Investigate different theories (gravity, cosmology)

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