Computational Cosmology Group
Of Shahid Beheshti University

Morphological measures as indicators for critical transitions in complex systems

معیارهای ریخت‌شناسی به عنوان نشانگرهای گذارهای بحرانی در سامانه‌های پیچیده

Nazanin Imannezhad

Supervisor:
Prof. S. M. S. Movahed

Complex systems ， critical transitions

- Many real-world systems like ecosystems, climate, biological systems are complex and are characterized by the ability to **self-organise** and **adapt** under external influences or environments. However, due to their **multi-component** nature, **nonlinear** behavior and **complex pattern of interactions**, they also show **multi-stability** that creates the possibility of sudden, unexpected shifts from one dynamical state to another.

- These abrupt transitions are referred to as **critical transitions**. Many of these transitions occur with tiny changes in intrinsic or extrinsic conditions and are difficult to anticipate.

- often, such transitions cause natural calamities or failures in engineering systems and infrastructure. Therefore, predicting their occurrence is essential to mitigate disaster impacts and to manage the risks involved.

Common Pipeline

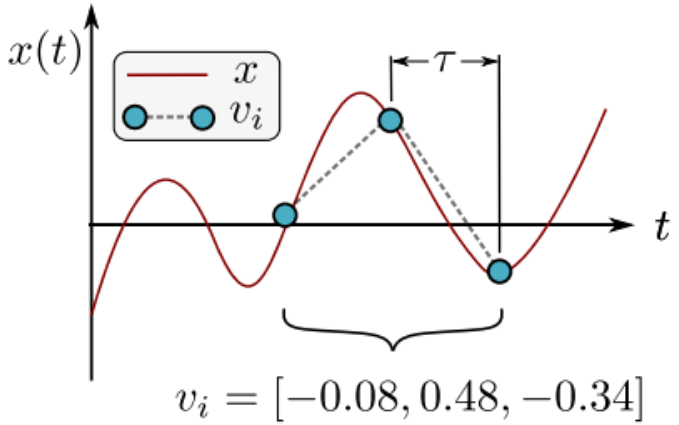
- **common pipeline for topological signal processing (TSP)**
  The standard pipeline for TSP constructs a filtration of simplicial complexes (called the Vietoris-Rips complex) based on point cloud data generated from the state-space reconstruction (SSR) of an input time series

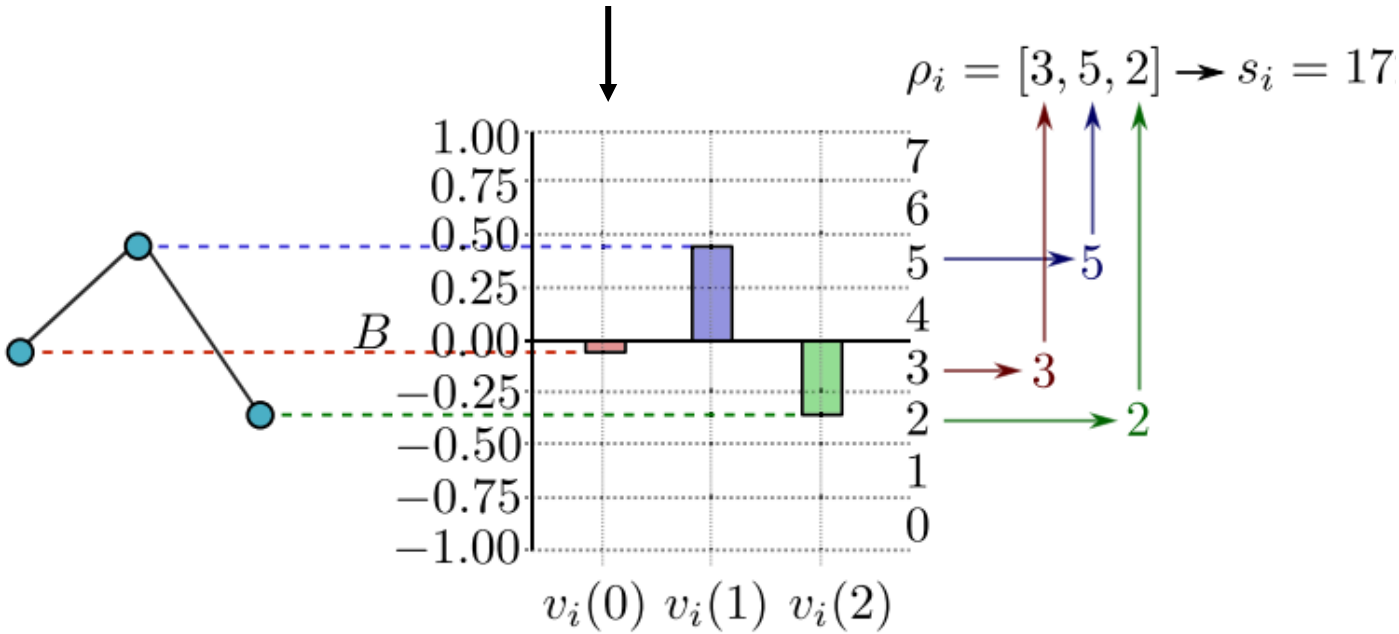
- **Problem**
  problem with this pipeline is its computational demand having complexity $O(N^3)$, where $N = \binom{n}{d+1}$ is the size of the simplicial complex with n as the number of points in the simplical complex and d as the maximum dimension of the used homology. For long signals, this makes this standard pipeline computationally infeasible.

- **Solution**
  analyzing time series via representations as **complex networks**.

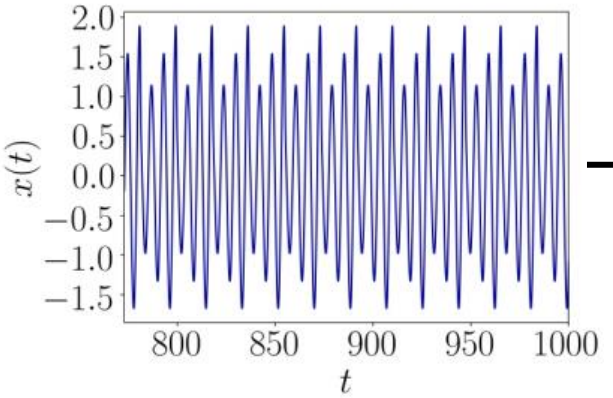
[A. D. Myers et al, *Persistent Homology of Coarse-Grained State-Space Networks*, Phys Rev E **107**, (2023)]

Coarse-Grained State-Space Networks

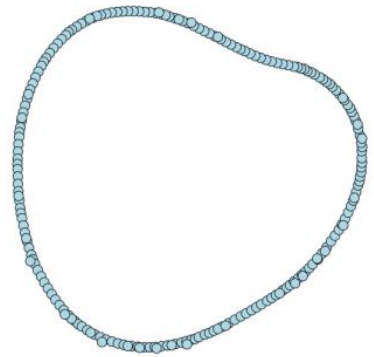




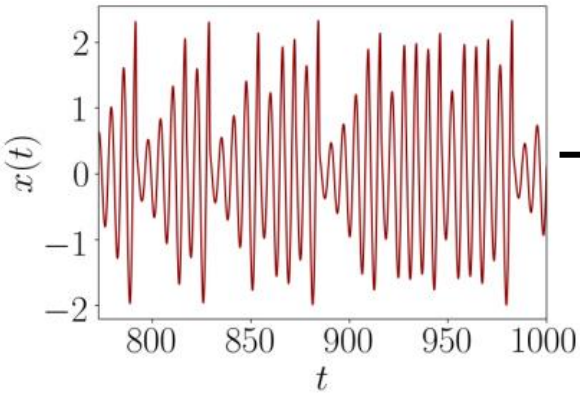
Periodic TS

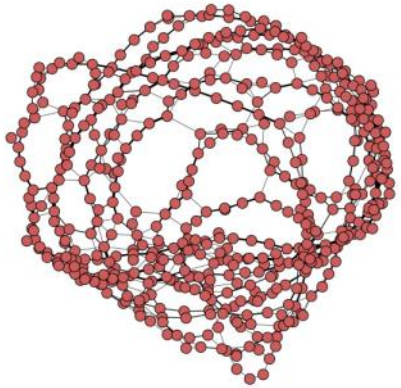


CGSS Network



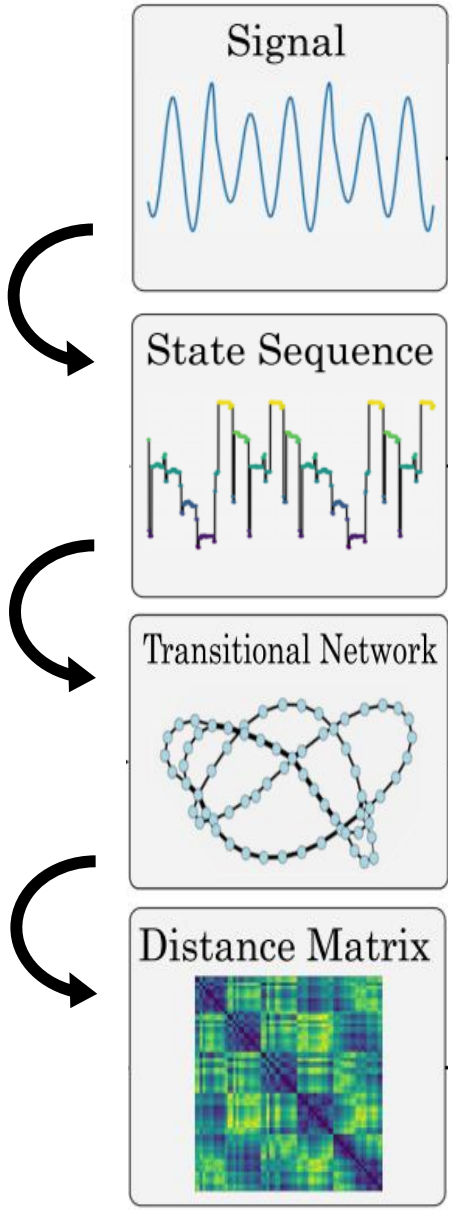
Chaotic TS

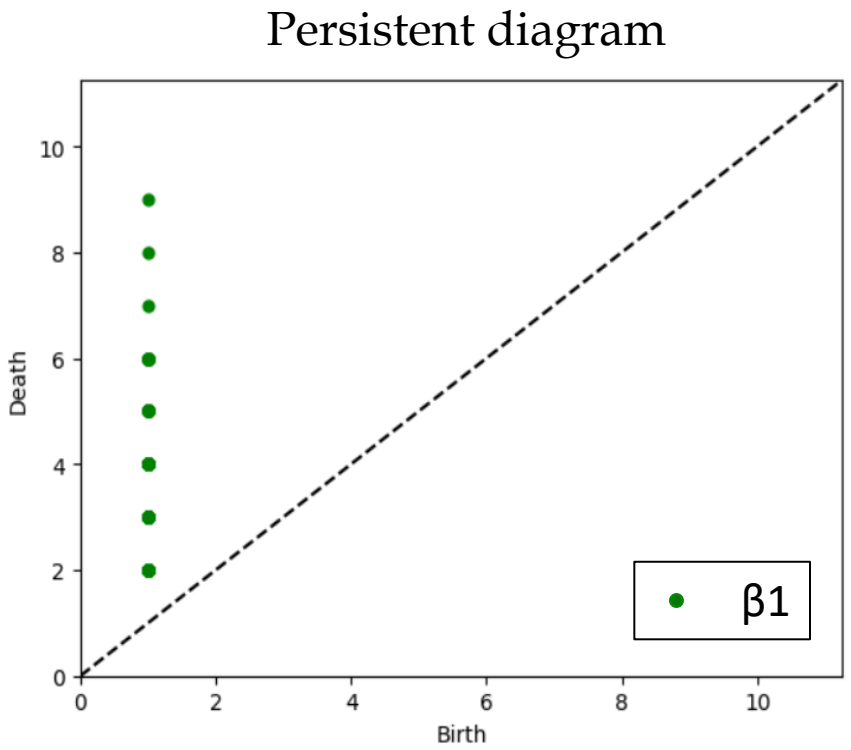




$s_i = 1 + \sum_{j=0}^{n-1} \rho_i(j) b^j = 1 + 3(8^0) + 5(8^1) + 2(8^2) = 172$

Alternative pipeline





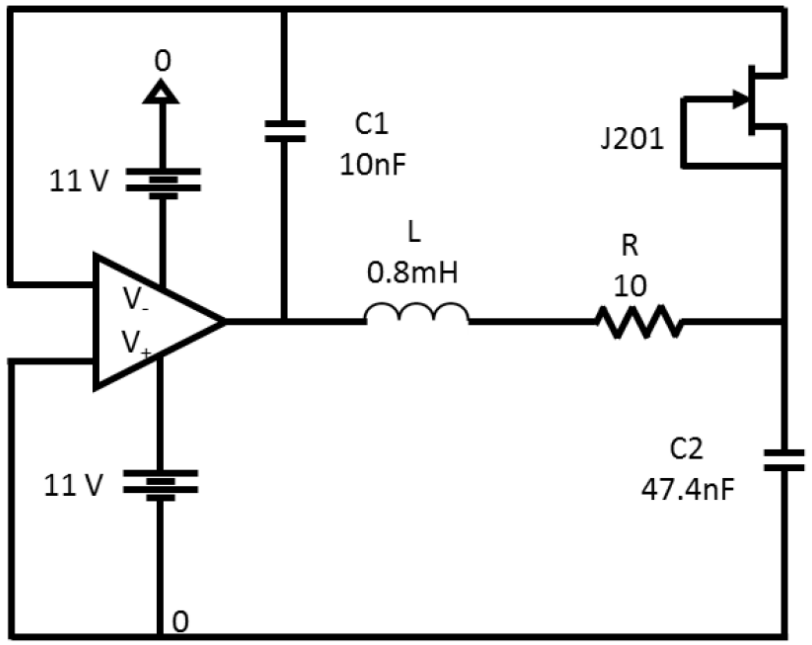
features

1. **Number of β1**
2. **Mean of death**
3. **Max o of death**

[A. D. Myers et al, *Persistent Homology of Coarse-Grained State-Space Networks*, Phys Rev E **107**, (2023)]

Jerk circuit dynamic system

- Op-amp based Jerk circuit which is **a real life continuous time system** that exhibits period-doubling cascade followed by chaotic behavior. This **non-linear dynamical system** composed of **three-dimensional state Vector**
  as follows:



$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\alpha \\ -\beta & \beta & -\beta\gamma \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 1 \\ \alpha \\ 0 \end{pmatrix}\varphi(y)$$

$$\varphi(y) = \begin{cases} -y & \text{if } y \le 1 \\ -1 & \text{if } y > 1 \end{cases}$$

[Mittal K, Gupta S. *Topological characterization and early detection of bifurcations and chaos in complex systems using persistent homology*. Chaos. (2017)]

AWGN (Additive white gaussian noise) with 30 dB SNR (Signal to noise rate)

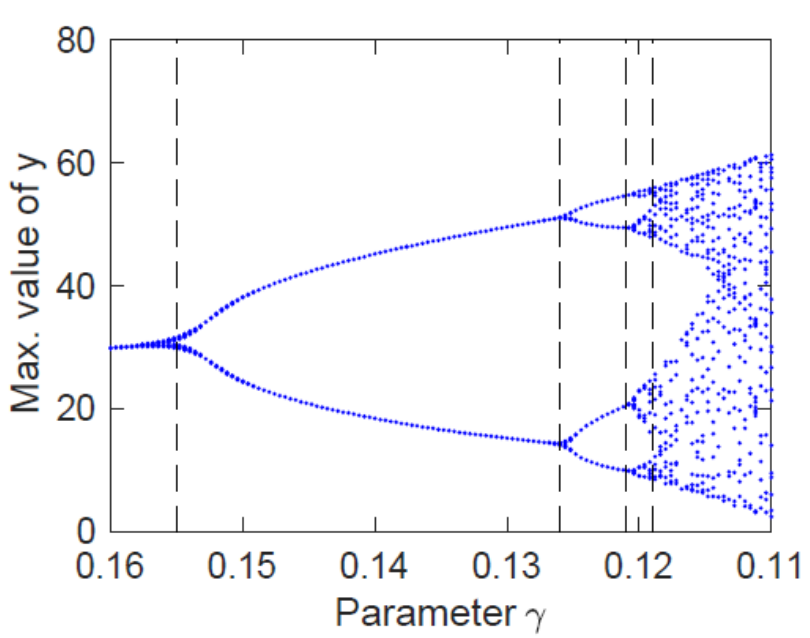
- I added AWGN with 30 dB SNR to my generated time series for realistic simulations according to bellow formula:

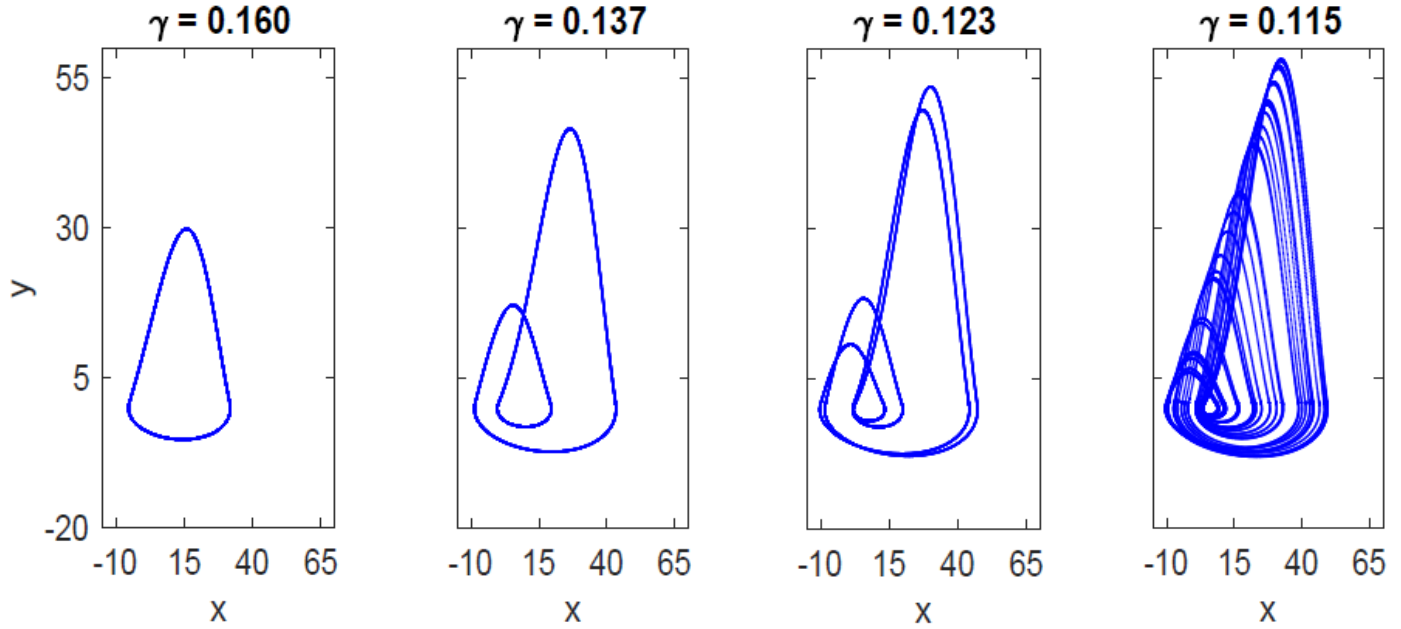
$$SNR = \frac{\mu_{signal}}{\sigma_{noise}}$$

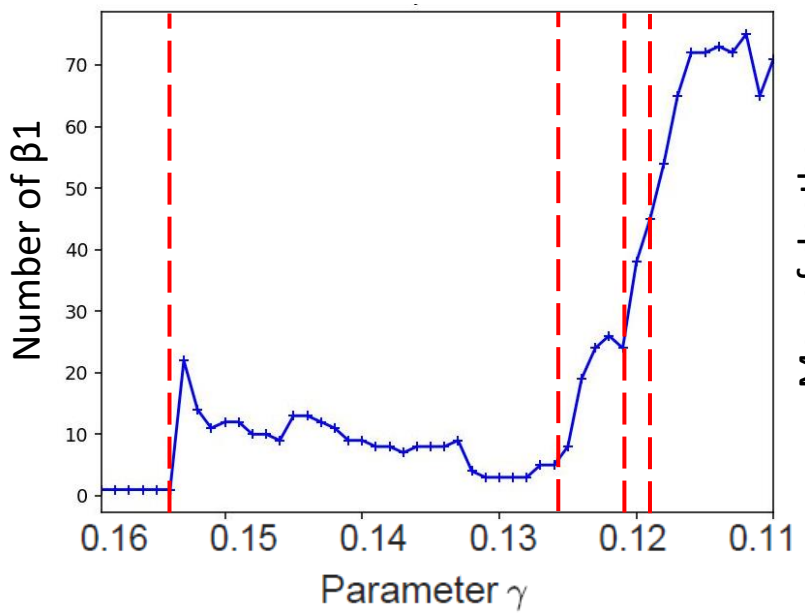
$$SNR_{dB} = 10log_{10}(SNR) = 30$$

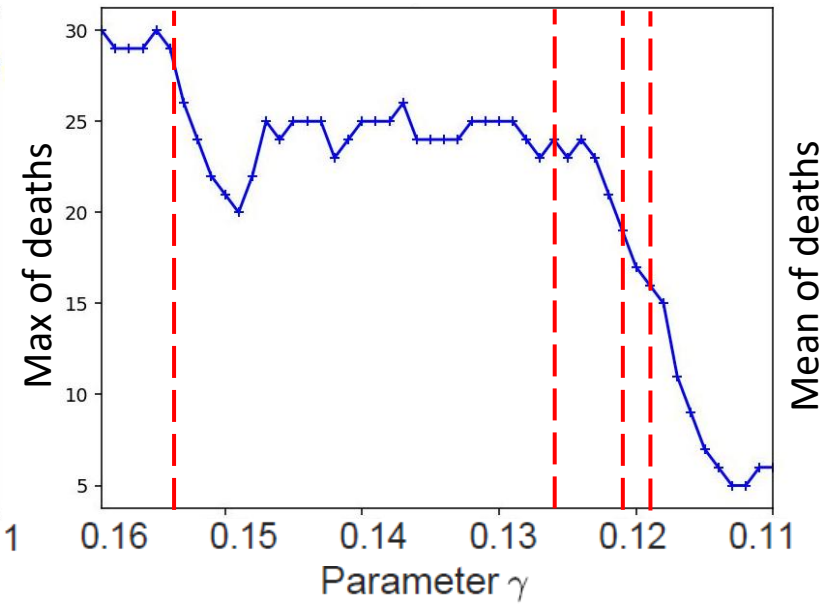
$$\sigma_{noise} = \frac{\mu_{signal}}{10^{SNR_{dB}/10}} = \frac{\mu_{signal}}{10^3}$$

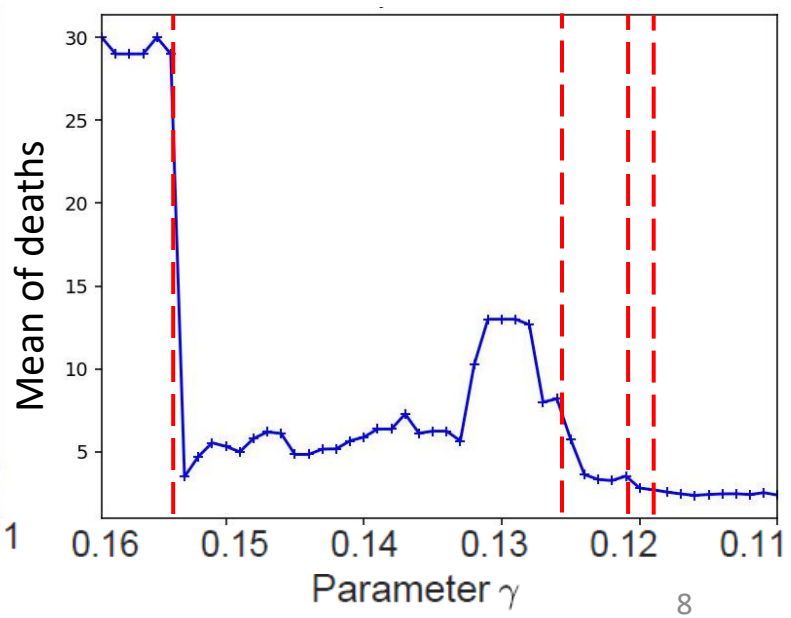
[https://en.wikipedia.org/wiki/Signal-to-noise_ratio]


Max. value of y
Parameter γ


γ = 0.160
γ = 0.137
γ = 0.123
γ = 0.115
y
x


Number of β1
Parameter γ


Max of deaths
Parameter γ


Mean of deaths
Parameter γ

THANK YOU

Gantt Chart

سال		سال 1402					سال 1403											
ردیف	عنوان فعالیت	آبان	آذر	دی	بهمن	اسفند	فروردین	اردیبهشت	خرداد	تیر	مرداد	شهریور	مهر	آبان	آذر	دی
۱	مطالعه‌ی نسبتاً جامع و تخصصی از مفاهیم گذار بحرانی، نقطهٔ عطف و آهستگی بحرانی															
۲	تحلیل مبتنی بر ریخت‌شناسی(هندسی و توپولوژی)															
۳	پیش‌پردازش مبتنی بر شبکه‌های پیچیده															
۳	مدلهای دینامیکی برای مطالعهٔ گذار بحرانی															
۴	جستجو برای یافتن ویژگی مناسب یافتن فاز و همچنین هشدار زودهنگام															
۵	اعمال روشهای ابداعی بر روی داده‌های واقعی															
۶	ساخت بردار ویژگی و ارزیابی ماشینی															
۷	تحلیل و برّرسی نتایج به دست آمده و ارزیابی اثرات کمیت‌های فیزیک بر روی توانایی شناسایی و هشدار زودهنگام از گذار بحرانی															
۸	ارائه گزارش نهایی و تهیه مقاله و نگارش پایان‌نامه															
۹	دفاع از پایان‌نامه															